MA109 — College Algebra Spring 2016
Exam 2 2016-03-09

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1. (a) (b) (c) (d) (e)
2. (a) (b) (c) (d) (e)
3. (a) (b) (c) (d) (e)
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15. (a) (b) (c) (d) (e)
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17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

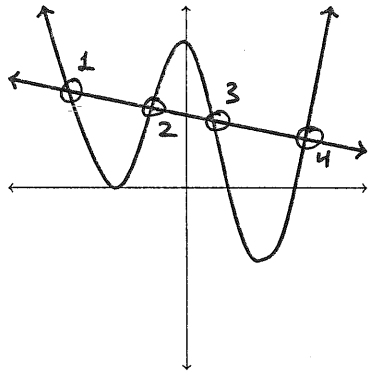
Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. The graph of two equations is shown below. Determine the number of solutions for the system of equations.

The two lines intersect at 4 points



Possibilities:

(a) 4

(b) 0

(c) 3

(d) 2

(e) 1

2. Use the substitution method to find all solutions of the system of equations.

$$\begin{aligned} x + y^2 &= 46 \\ y - x &= -4 \end{aligned}$$

$y + 4 = x$

$\Rightarrow x + y^2 = 46$ $\Rightarrow y + 4 + y^2 = 46$

$\Rightarrow y^2 + y - 42 = 0$

$\Rightarrow (y+7)(y-6) = 0$

$\Rightarrow y = -7$ or $y = 6$

$y + 4 = x$
$\Rightarrow -7 + 4 = x$
$\Rightarrow x = -3$

$(x = -3, y = -7)$

$y + 4 = x$
$\Rightarrow 6 + 4 = x$
$\Rightarrow x = 10$

$(x = 10, y = 6)$

Possibilities:

(a) $(x = 10, y = 6)$ only

(b) $(x = -3, y = -7)$ only

(c) $(x = -3, y = 7)$ and $(x = 11, y = -7)$

(d) $(x = -3, y = -7)$ and $(x = 10, y = 6)$

(e) No real solutions

3. Suppose you are solving the system of equations below using the substitution method. You solve for y in the first equation and substitute it into the second equation. What equation must you solve then?

$$\begin{aligned} 8x^7 + y &= 5 \\ 4x^9 + 3y &= 6 \end{aligned}$$

$y = 5 - 8x^7$

$\Rightarrow 4x^9 + 3y = 6$

$\Rightarrow 4x^9 + 3(5 - 8x^7) = 6$

Possibilities:

(a) $4\left(\sqrt[7]{5-8x^7}\right)^9 + 3y = 6$

(b) $4x^9 + 3\left(\sqrt[7]{5-y}\right) = 6$

(c) $4x^9 + 3(5 - 8x^7) = 6$

(d) $4x^9 + 3\left(\sqrt[7]{5-8x^7}\right) = 6$

(e) $4(5 - 8x^7)^9 + 3y = 6$

4. Use the elimination method to solve the system. The multiple choice problem only asks you for y.

$$\begin{array}{rrcrcl} & 29x & + & 17y & = & 218 \\ - & 29x & + & 16y & = & 212 \\ \hline & 0x & + & 1y & = & 6 \end{array}$$

$\Rightarrow y = 6$

Subtract the bottom equation from the top equation.

$29x - 29x = 0x$ $\quad 17x - 16x = 1x$ $\quad 218 - 212 = 6$

Possibilities:

(a) Every solution has $y = \dfrac{6}{29}$

(b) Every solution has $y = 6$

(c) Every solution has $y = 4$

(d) Every solution has $y = -17$

(e) Every solution has $y = \dfrac{6}{17}$

5. Use substitution to solve the system.

$$\begin{aligned} 5x &+ 3y = 13 \\ 4x &+ 2y = 29 \end{aligned}$$

$5x + 3y = 13 \Rightarrow 3y = 13 - 5x \Rightarrow y = \frac{13}{3} - \frac{5}{3}x$

Possibilities:

(a) $x = -\frac{93}{2}, y = \frac{61}{2}$

(b) $x = 152, y = 110$

(c) $\boxed{x = \frac{61}{2}, y = -\frac{93}{2}}$

(d) $x = -\frac{1}{76}, y = -\frac{1}{55}$

(e) $x = \frac{13}{5}, y = \frac{29}{4}$

Use x to solve for y:

$x = \frac{61}{2}$

$\Rightarrow y = \frac{13}{3} - \frac{5}{3}x$

$\Rightarrow y = \frac{13}{3} - \frac{5}{3}\left(\frac{61}{2}\right)$

$\Rightarrow y = \frac{26}{6} - \frac{305}{6}$

$\Rightarrow y = -\frac{279}{6}$

$\Rightarrow \boxed{y = -\frac{93}{2}}$

$4x + 2y = 29 \Rightarrow 4x + 2\left(\frac{13}{3} - \frac{5}{3}x\right) = 29$

$\Rightarrow 4x + \frac{26}{3} - \frac{10}{3}x = 29$

$\Rightarrow \frac{12x}{3} - \frac{10x}{3} + \frac{26}{3} = 29$

$\Rightarrow \frac{2x}{3} + \frac{26}{3} = 29$

$\Rightarrow \frac{2x}{3} = \frac{87}{3} - \frac{26}{3}$

$\Rightarrow \frac{2x}{3} = \frac{61}{3} \Rightarrow x = \frac{61}{3} \cdot \frac{3}{2} \Rightarrow \boxed{x = \frac{61}{2}}$

multiply both sides by $\frac{3}{2}$

6. Use the elimination method to find all solutions of the system of equations.

$$\begin{cases} \frac{37}{x} + \frac{22}{y} = 123 \\ \frac{17}{x} + \frac{11}{y} = 58 \end{cases}$$

Steps:
1.) Multiply the bottom equation by 2
2.) Subtract the bottom equation from the top
3.) Solve for x
4.) Plug in new found value of x into one of the equations
5.) Solve for y.

Possibilities:

(a) $(x = 37, y = 22)$, $(x = -37, y = 22)$, $(x = 17, y = 11)$, and $(x = -17, y = -11)$

(b) $\boxed{\left(x = \frac{3}{7}, y = \frac{3}{5}\right) \text{ only}}$

(c) $(x = 37, y = 22)$ and $(x = 17, y = 11)$

(d) $(x = -37, y = 22)$ and $(x = -17, y = 11)$

(e) $\left(x = \frac{3}{7}, y = \frac{3}{5}\right)$ and $\left(x = -\frac{3}{7}, y = -\frac{3}{5}\right)$

$\frac{37}{x} + \frac{22}{y} = 123$

$- \frac{34}{x} + \frac{22}{y} = 116$

$\frac{3}{x} + \frac{0}{y} = 7$

$\Rightarrow \frac{3}{x} = 7$

$\Rightarrow \boxed{x = \frac{3}{7}}$

$\Rightarrow \frac{37}{x} + \frac{22}{y} = 123$

see * below

$\frac{37}{3/7} + \frac{22}{y} = 123$

$\Rightarrow \frac{259}{3} + \frac{22}{y} = 123$

$\Rightarrow \frac{22}{y} = \frac{369}{3} - \frac{259}{3}$

$\Rightarrow \frac{22}{y} = \frac{110}{3}$

$\Rightarrow \frac{22}{1} \cdot \frac{3}{110} = y$

$\Rightarrow \frac{2}{1} \cdot \frac{3}{10} = y$

$\Rightarrow \boxed{y = \frac{3}{5}}$

* $\frac{37}{3/7} = \frac{37}{1} \div \frac{3}{7} = \frac{37}{1} \cdot \frac{7}{3} = \frac{37 \cdot 7}{3} = \frac{259}{3}$

Dividing by a fraction is the same as multiplying by its reciprocal

7. Use algebraic, graphical, or numerical methods to find all real solutions of the equation, approximating when necessary to four decimal places.

$$\frac{3x}{x-20} = 5$$

Possibilities:

(a) $x = 50.0000$

(b) $x = 50.1317$

(c) $x = 50.2634$

(d) $x = 50.3951$

(e) $x = 50.5268$

- Plot $y = \frac{3x}{x-20}$ and $y = 5$ on the same graph.
- Zoom in around $x = 50$, and see where $y = 5$ intersects $y = \frac{3x}{x-20}$
- You can also notice that plugging in $x = 50$ makes $\frac{3x}{x-20}$ equal to 5.

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8. Find an equation that helps solve for the worker's old salary, call it x, in the following problem:

A worker gets a 5.35% pay raise and now makes \$1350 per month. What was the worker's old salary?

Possibilities:

(a) $x = 1350 - 5.35$

(b) $5.35x = 1350$

(c) $x = (5.35)(1350)$

(d) $0.0535x = 1350$

(e) $x + 0.0535x = 1350$

In words:

Take the worker's old salary (x), add to that 5.35% of the old salary (which is $0.0535x$) and that equals \$1350.

* Rember: To get a percentage of a value, multiply that value by a decimal where the decimal point is moved two digits to the left

As an equation:

$x + 0.0535x = 1350$

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9. You have already invested \$700 in a stock with an annual return of 10%. How much of an additional \$1,250 should be invested at 20% and how much at 5% so that the total return on the entire \$1,950 is 15%?

What equations should be solved if x is the amount of money invested at 20% and y is the amount of money invested at 5%?

Initial amounts have to add up to \$1950

$700 + 1250 = 1{,}950 \Rightarrow \boxed{700 + x + y = 1950}$

$x+y$: Portion of \$1250 invested at 20% / Portion of \$1250 invested at 5%

Amounts earned from interest must add up to 15% of whole amount invested (\$1950)

- 10% of 700 $= 0.10(700)$
- 20% of amount invested at 20% $= 0.20x$
- 5% of amount invested at 5% $= 0.05y$

① sum of ① equals ②

- 15% of whole amount invested $= 0.15(1950)$ ②

$\boxed{0.10(700) + 0.20x + 0.05y = 0.15(1950)}$

Possibilities:

(a) $\left\{ \begin{array}{rcl} x &=& .20(1250) \\ y &=& .05(700) \end{array} \right\}$

(b) $\left\{ \begin{array}{rcl} .05x + .20y &=& .10(1250) \\ .20x + .05y &=& .15(1950) \end{array} \right\}$

(c) $\left\{ \begin{array}{rcl} x + y &=& 700 \\ .20x + .05y &=& .15(1950) \end{array} \right\}$

(d) $\left\{ \begin{array}{rcl} 700 + x + y &=& 1950 \\ .10(700) + .20x + .05y &=& .15(1950) \end{array} \right\}$

(e) $\left\{ \begin{array}{rcl} .15 + x + y &=& 1950 \\ 1250 + .20x + .05y &=& .10(700) \end{array} \right\}$

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10. A concrete walk of uniform width is to be built around a giant circular pool. The radius of the pool is 18 meters, and enough concrete is available to cover 64.09π square meters (approximately). If all the concrete is to be used, how wide should the walk be (approximately)? Choose the closest answer.

Possibilities:

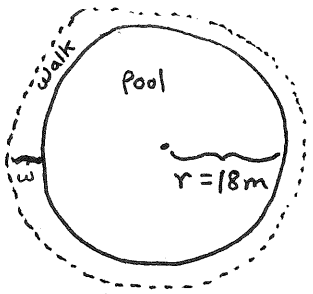
(a) 46.1 meters wide

(b) 18 meters wide

(c) 9.99 meters wide

(d) 1.7 meters wide

(e) 3.56 meters wide



let w = width of the walk

r = radius of pool

$r+w$ = radius of pool + walk

area of pool $= \pi r^2$

area of pool + walk $= \pi(r+w)^2 = \pi(r^2+2rw+w^2)$

area of walk = area of pool + walk − area of pool

$= \pi(r+w)^2 - \pi r^2$

$= \pi(r^2+2rw+w^2) - \pi r^2$

$= \pi r^2 + 2\pi rw + \pi w^2 - \pi r^2$

$= 2\pi rw + \pi w^2$

$2\pi(18)w + \pi w^2 = 64.09\pi$

$\Rightarrow 36w + w^2 = 64.09$

$\Rightarrow w^2 + 36w - 64.09 = 0$

* graphing $x^2+36x-64.09$ we see it intersects the x-axis at around $x = 1.7$ and $x = -37.7$

width is positive so $w = x = 1.7$ is the answer

11. Find the equilibrium price. In the supply and demand equations, p is price (in dollars) and x is quantity (in thousands). Please round your answer to the nearest hundredth (the nearest cent).

$$\begin{aligned} \text{Supply: } p &= 8x - 7 \\ \text{Demand: } p &= -4x + 9 \end{aligned}$$

Possibilities:

(a) $p = \$1.33$

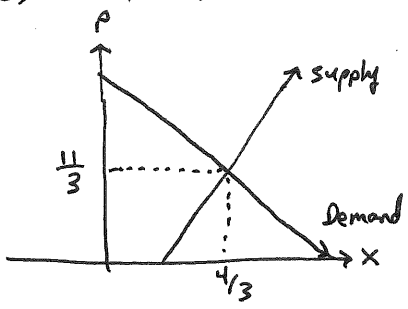
(b) $p = \$3.66$

(c) $p = \$4$

(d) $p = \$6$

(e) $p = \$2$

Equilibrium price is the value of p (price) which makes supply equal to demand.



supply = demand : $8x - 7 = -4x + 9$

$\Rightarrow 12x = 16$

$\Rightarrow x = \frac{16}{12} = \frac{4}{3}$

$\Rightarrow p = 8\left(\frac{4}{3}\right) - 7$

$\Rightarrow p = \frac{32}{3} - \frac{21}{3}$

$\Rightarrow p = \frac{11}{3} \approx 3.66$

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12. A radiator contains 4 quarts of fluid, 25% of which is antifreeze. How much fluid should be drained and replaced with pure (100%) antifreeze so that the new mixture is 65% antifreeze?

Possibilities:

(a) 2.13 quarts drained and replaced

(b) 2.6 quarts drained and replaced

(c) 1 quarts drained and replaced

(d) 6.4 quarts drained and replaced

(e) 4 quarts drained and replaced

x = amount that will be drained and replaced with 100% antifreeze mixture (in quarts)

amt of antifreeze to begin with: $4(.25) = 1$ quart

amt of antifreeze after removing x: $(4-x)(.25)$

amt of antifreeze after adding x: $(4-x)(.25)+x$ } set equal, solve for x

amt of antifreeze we want: $4(.65)$

Solve for x:

$(4-x)(.25)+x = 4(.65)$ → $4\left(\frac{65}{100}\right) = \frac{65}{25} = \frac{13}{5}$

$\Rightarrow 1 - 0.25x + x = \frac{13}{5}$

$\Rightarrow 1 + 0.75x = \frac{13}{5}$

$\Rightarrow \frac{3}{4}x = \frac{13}{5} - 1 = \frac{8}{5}$

$\Rightarrow x = \frac{8}{5} \cdot \frac{4}{3} = \frac{32}{15} \approx$ **2.13 quarts**

13. Solve the inequality. Answer in interval notation.

$6x + 18 \leq 5x + 20$

$6x+18 \leq 5x+20$
$-5x\ -18 \quad -5x\ -18$
$x \leq 2$
$\Rightarrow x \leq 2$

Possibilities:

(a) $[2, \infty)$

(b) $[3, 4]$

(c) $(-\infty, 2]$

(d) $(-\infty, \infty)$

(e) $(-\infty, 6] \cup [18, \infty)$

14. A business executive is considering two options for leasing a car. The first option is \$355 per month, but the first month costs \$90 extra. The second option is \$370 per month with no extra cost for the first month. The business executive wants to know which option is cheapest based on how many months they plan on leasing the car. Which choice below most accurately describes the situation?

1st option: $P_1 = 355x + 90$
2nd option: $P_2 = 370x$
equilibrium point:
$355x + 90 = 370x$
$\Rightarrow 90 = 370x - 355x$
$\Rightarrow 90 = 15x$
$\Rightarrow x = 6$
(x is the number of months)

Possibilities:

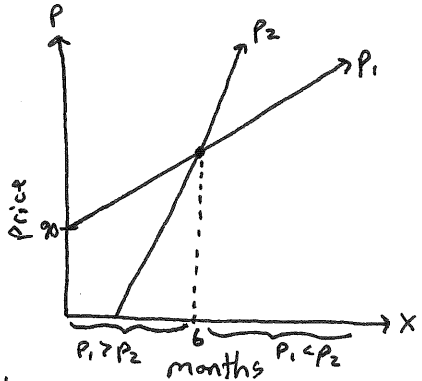
(a) The first option is cheaper if the lease is 14 months or shorter, the second option is cheaper if the lease is 16 months or longer, and the two options are the same price if the lease is exactly 15 months

(b) The first option is cheaper if the lease is 16 months or longer, the second option is cheaper if the lease is 14 months or shorter, and the two options are the same price if the lease is exactly 15 months

(c) Both options cost the same regardless of the length of the lease.

(d) The first option is cheaper if the lease is 7 months or longer, the second option is cheaper if the lease is 5 months or shorter, and the two options are the same price if the lease is exactly 6 months

(e) The first option is cheaper if the lease is 5 months or shorter, the second option is cheaper if the lease is 7 months or longer, and the two options are the same price if the lease is exactly 6 months



* P_2 has a larger slope than P_1 this helps us graph the two lines

* when $x < 6$ then P_1 line is above P_2 line

* when $x > 6$ then P_1 line is below P_2 line

15. Solve the inequality. Express your answer in interval notation.

$$\left|\frac{8}{9}+\frac{1}{9}x\right| \leq \frac{4}{9}$$

Possibilities:

(a) $[0, 12]$

(b) $[\frac{4}{9}, \frac{8}{9}]$

(c) $[4, 12]$

(d) $[0, \frac{4}{9}]$

(e) $[-12, -4]$

$\Rightarrow -\frac{4}{9} \leq \frac{8}{9}+\frac{1}{9}x \leq \frac{4}{9}$

$-\frac{8}{9} \quad -\frac{8}{9} \quad -\frac{8}{9}$

$\Rightarrow \frac{-12}{9} \leq \frac{1}{9}x \leq -\frac{4}{9}$

$\Rightarrow \left(-\frac{4}{3} \leq \frac{1}{9}x \leq -\frac{4}{9}\right) \cdot 9$

$\Rightarrow -\frac{4\cdot 9}{3} \leq \frac{1}{9}x \cdot 9 \leq -\frac{4\cdot 9}{9}$

$\Rightarrow \boxed{-12 \leq x \leq -4}$

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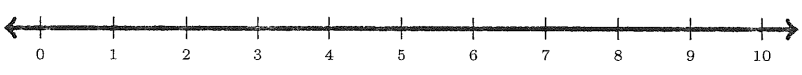
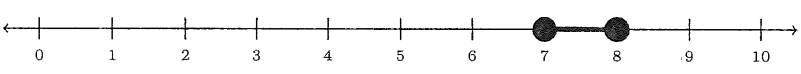
16. Solve the inequality. Answer by choosing the correct number line.

$0 \leq (x-8)(x-7)^2$

Possibilities:

(a)


(b)


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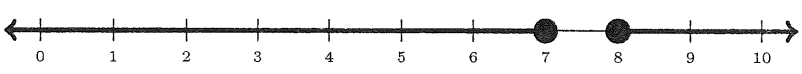

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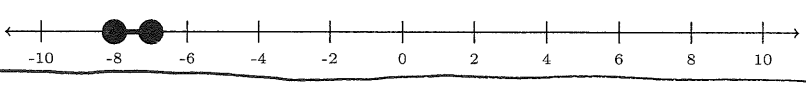

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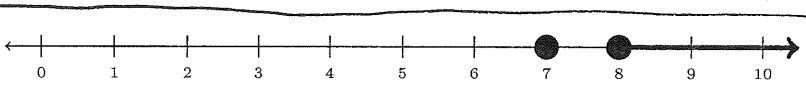


- Find x values where right hand side is equal to zero:

$(x-8)(x-7)^2=0$

$\Rightarrow (x-8)=0$ or $(x-7)^2=0$

$\Rightarrow x=8$ or $x-7=0$

$\Rightarrow x=8$ or $x=7$

- Make a sign chart

test point: $x=6$

$(6-8)(6-7)^2=-2(1)<0$

$x=7.5$

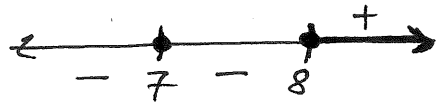
$(7.5-8)(7.5-7)^2$

$=-0.5(0.5)^2<0$

$x=9$

$(9-8)(9-7)^2=1(2)^2>0$

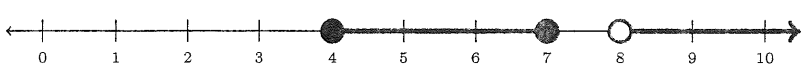
Remember $\leq$ means include the x-values which make $(x-8)(x-7)^2=0$, i.e. $x=7$ and $x=8$. Including these values means using filled in dots on the number line.



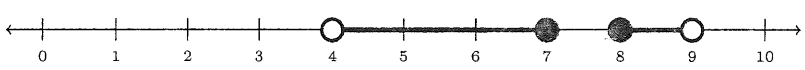
17. Solve the inequality. Answer by choosing the correct number line.

$$0 \leq \frac{(x-8)(x-7)}{(x-4)(x-9)}$$

Possibilities:

(a)


(b)


(c)


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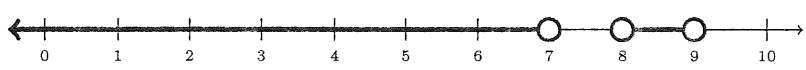
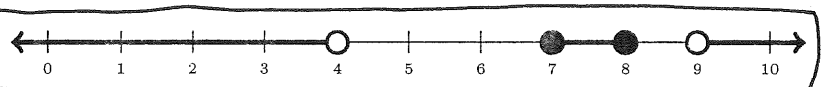


(e)


$\star = \frac{(x-8)(x-7)}{(x-4)(x-9)}$ → zeros of numerator

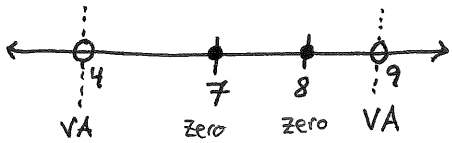
$\star = 0 \Leftrightarrow (x-8)(x-7) = 0$

$\Leftrightarrow x = 7$ or $x = 8$

vertical asymptotes of $\star$:

= zeros of denominator

= $x = 4$ and $x = 9$



test points

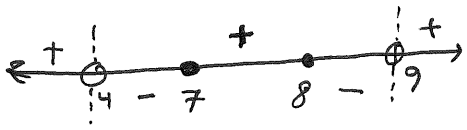
$x = 3$: $\frac{(3-8)(3-7)}{(3-4)(3-9)} = \frac{(-)(-)}{(-)(-)} = (+)$

$x = 5$: $\frac{(5-8)(5-7)}{(5-4)(5-9)} = \frac{(-)(-)}{(+)(-)} = \frac{(+)}{(-)} = (-)$

$x = 7.5$: $\frac{(7.5-8)(7.5-7)}{(7.5-4)(7.5-9)} = \frac{(-)(+)}{(+)(-)} = \frac{(+)}{(+)}$

$x = 8.5$: $\frac{(8.5-8)(8.5-7)}{(8.5-4)(8.5-9)} = \frac{(+)(+)}{(+)(-)} = \frac{(+)}{(-)} = (-)$

$x = 10$: $\frac{(10-8)(10-7)}{(10-4)(10-9)} = \frac{(+)(+)}{(+)(+)} = \frac{(+)}{(+)} = (+)$



18. Find $f(-3)$ from the graph of $y = f(x)$.

Possibilities:

(a) $f(-3) = -2$

(b) $f(-3) = 0$

(c) $f(-3) = -4$

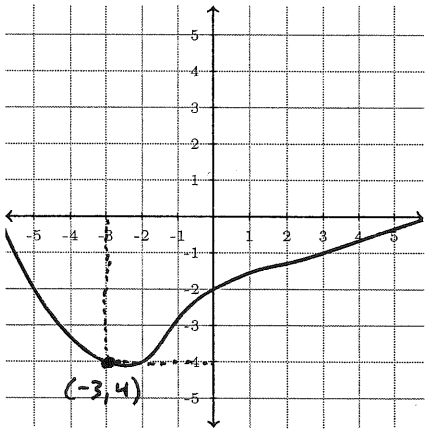
(d) $f(-3) = 7$

(e) $f(-3) = -3$

$f(-3) = -4$

↑ x-value ↑ y-value

Draw a vertical line at $x=-3$ and see where it hits the graph of $f(x)$. The point is $(-3,-4)$ so the y-value is -4.



19. Find the indicated value of the function when $x = \sqrt{3} + 7$.

$$f(x) = \sqrt{x+4} - x - 9$$

$f(\sqrt{3}+7) =$

Possibilities:

(a) $\sqrt{\sqrt{3}+11} - \sqrt{3} - 16$

(b) $\sqrt{11} - 16$

(c) $\sqrt{\sqrt{3}+11} - \sqrt{3} - 2$

(d) -5

(e) $\sqrt{14} - \sqrt{3} - 16$

$f(x) = \sqrt{(\sqrt{3}+7)+4} - (\sqrt{3}+7) - 9$

$= \sqrt{\sqrt{3}+11} - \sqrt{3} - 7 - 9$

$= \sqrt{\sqrt{3}+11} - \sqrt{3} - 16$

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20. Let $f(x) = 2x^2 + 8$. Find $\dfrac{f(x+h) - f(x)}{h}$ if $h \neq 0$. Simplify your answer.

Possibilities:

(a) $\dfrac{2h^2+8}{h}$

(b) $4x + 2h$

(c) 16

(d) $2x + 8h$

(e) $\dfrac{h+16}{h}$

$f(x) = 2x^2 + 8$

$f(x+h) = 2(x+h)^2 + 8$

$= 2(x^2 + 2xh + h^2) + 8$

$= 2x^2 + 4xh + 2h^2 + 8$

$$\frac{f(x+h) - f(x)}{h} = \frac{2x^2+4xh+2h^2+8-(2x^2+8)}{h}$$

$$= \frac{2x^2+4xh+2h^2+8-2x^2-8}{h}$$

$$= \frac{4xh+2h^2}{h} = \frac{(4x+2h)h}{h} = 4x+2h$$
