MA109 — College Algebra Spring 2016
Exam 3 2016-03-13

Name: _____ Sec.: _____

Do not remove this answer page — you will turn in the entire exam. You have two hours to do this exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of multiple choice questions. Record your answers on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

Do not circle answers on this page, but please circle the letter of each correct response in the body of the exam. It is your responsibility to make it CLEAR which response has been chosen. You will not get credit unless the correct answer has been marked on both this page and in the body of the exam.

GOOD LUCK!

1.	a	b	c	**d**	e	11.	a	**b**	c	d	e
2.	**a**	b	c	d	e	12.	a	**b**	c	d	e
3.	**a**	b	c	d	e	13.	a	**b**	c	d	e
4.	**a**	b	c	d	e	14.	a	b	c	**d**	e
5.	a	b	c	d	**e**	15.	a	b	c	d	**e**
6.	a	b	**c**	d	e	16.	a	b	**c**	d	e
7.	a	b	c	**d**	e	17.	a	b	c	d	**e**
8.	a	**b**	c	d	e	18.	**a**	b	c	d	e
9.	a	b	c	d	**e**	19.	a	b	c	**d**	e
10.	a	b	**c**	d	e	20.	a	b	**c**	d	e

For grading use:

Number Correct	
	(out of 20 problems)

Total	
	(out of 100 points)

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

1. Find the indicated value of the function when $x = \sqrt{3} + 7$.

$$f(x) = \sqrt{x+6} - x - 5$$

$f(\sqrt{3}+7) = \sqrt{(\sqrt{3}+7)+6} - (\sqrt{3}+7) - 5 = \sqrt{\sqrt{3}+13} - \sqrt{3} - 7 - 5$

$= \sqrt{\sqrt{3}+13} - \sqrt{3} - 12$

Possibilities:

(a) $\sqrt{\sqrt{3}+13} - \sqrt{3} - 2$

(b) $\sqrt{16} - \sqrt{3} - 12$

(c) $\sqrt{13} - 12$

(d) $\sqrt{\sqrt{3}+13} - \sqrt{3} - 12$

(e) 1

2. Find $f(4)$ if $f(x) = \begin{cases} 6 & \text{if } x \leq 1 \\ 2x+4 & \text{if } 1 < x \leq 3 \\ 3x+1 & \text{if } 3 < x \leq 5 \\ 16 & \text{if } x > 5 \end{cases}$ $3 < 4 \leq 5$ so we use $3x+1 = 3(4)+1 = 13$

Possibilities:

(a) 13

(b) 6

(c) 16

(d) 10

(e) 12

3. Find the domain of $\sqrt[3]{\frac{x-5}{4}}$

Possibilities:

(a) $(-\infty, \infty)$

(b) $(5, \infty)$

(c) $(-\infty, 5) \cup (5, \infty)$

(d) $[\frac{5}{4}, \infty)$

(e) $[5, \infty)$

There are no variables in the denominator of a fraction and there are no even roots that we have to worry about.

4. Find the domain of $\frac{5}{\sqrt{x-4}}$

Possibilities:

(a) $(4, \infty)$

(b) $[4, \infty)$

(c) $(-\infty, 4) \cup (4, \infty)$

(d) $[\frac{5}{4}, \infty)$

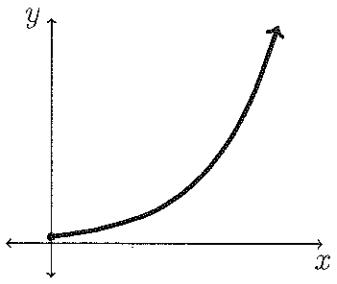
(e) $(-\infty, \infty)$

1.) denominator $\neq 0 \Rightarrow \sqrt{x-4} \neq 0 \Rightarrow x-4 \neq 0$
$\Rightarrow x \neq 4$

2.) expression under the radical is non-negative:
$x-4 \geq 0 \Rightarrow x \geq 4$

Combining 1 and 2 we get $x \geq 4$ or $x \in (4, \infty)$

5. Which situation below is most reasonably depicted in this graph:



Possibilities:

(a) y is the amount of water in a bucket at time x if a hole is made in the bucket at time $x = 0$.

(b) y is the how many days are left in the semester after x weeks of school, if $x = 0$ is the first week of class.

(c) y is the distance from home at time x as you run to the end of the block and back at a steady pace.

(d) y is the temperature of left-over food at time x if the food is placed in the refrigerator at time $x = 0$.

(e) y is the number of bacteria at time x if the bacteria experience a steady rate of exponential growth.

(a) NO. The graph would be decreasing
(b) NO. Same as (a)
(c) No. The graph would rise then fall
(d) No. Same as (a)
(e) Yes.

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6. A car moves along a straight test track. The distance traveled by the car at various times is shown in the table. Find the average speed of the car from 20 to 25 seconds.

Time (seconds)	0	5	10	15	20	25	30
Distance (feet)	0	50	200	450	800	1250	1800

Possibilities:

(a) 40 feet per second

(b) 60 feet per second

(c) 90 feet per second

(d) 100 feet per second

(e) 50 feet per second

$$\text{Avg speed} = \frac{\text{distance traveled}}{\text{time elapsed}} = \frac{1250-800}{25-20}$$

$$= \frac{450}{5} = 90 \text{ ft/sec}$$

7. Simplify the formula for the average rate of change of $f(x) = (x-5)^2 + 4$ from $x = 5$ to $x = 5 + h$

Possibilities:

(a) $5 + 2h$

(b) $2h$

(c) $10 + h$

(d) h

(e) 1

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h-5)^2+4-[(x-5)^2+4]}{h}$$

$$= \frac{(x+h)^2-10(x+h)+25+4-(x-5)^2-4}{h}$$

$$= \frac{x^2+2xh+h^2-10x-10h+25+4-x^2+10x-25-4}{h}$$

$$= \frac{2xh+h^2-10h}{h} = \frac{h(2x+h-10)}{h}$$

$$= 2x+h-10$$

$$\Rightarrow 2(5)+h-10 = h$$

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8. Find the domain of $\left(\frac{f}{g}\right)(x)$ if $f(x) = 2x^2 + 4x + 9$ and $g(x) = 5x - 8$

Possibilities:

(a) $(-\infty, \frac{5}{8})$

(b) $(-\infty, \frac{8}{5}) \cup (\frac{8}{5}, \infty)$

(c) $[\frac{8}{5}, \infty)$

(d) $(-\infty, \infty)$

(e) $\left[\frac{-4 \pm \sqrt{4^2-4(2)(9)}}{4}, \infty\right)$

domain = the set of all numbers where $\frac{f}{g}$ is defined

= the set of all numbers where $g(x) \neq 0$

= all x such that $5x - 8 \neq 0$

= all x such that $x \neq \frac{8}{5}$

$= (-\infty, \frac{8}{5}) \cup (\frac{8}{5}, \infty)$

9. Find $(f-g)(9)$ where $f(x) = 6x^2 - 5x - 3$ and $g(x) = 7x - 2$

Possibilities:

(a) 373

(b) 3064

(c) 22018

(d) 499

(e) 377

$f(9) = 6(9)^2 - 5(9) - 3 = 6(81) - 45 - 3 = 486 - 48 = 438$

$g(9) = 7(9) - 2 = 63 - 2 = 61$

$(f-g)(9) = f(9) - g(9) = 438 - 61 = 377$

10. Simplify the formula for $(f \circ g)(x)$ if $f(x) = 1 - x$ and $g(x) = \frac{x-1}{x}$

Hint: try plugging in $x = 13$

Possibilities:

(a) $13x$

(b) $\frac{13}{x}$

(c) $\frac{1}{x}$

(d) $\frac{x}{x-1}$

(e) x

$(f \circ g)(x) = f(g(x)) = f\left(\frac{x-1}{x}\right) = 1 - \left(\frac{x-1}{x}\right)$

$= \frac{x-(x-1)}{x} = \frac{1}{x}$

11. Suppose that the graph of $y = f(x)$ contains the point $(6, 5)$. Find a point that must be on the graph of $y = g(x)$ for $g(x) = 3 + f(7x + 2)$.

Possibilities:

(a) $(44, 2)$

(b) $\left(\frac{4}{7}, 8\right)$

(c) $\left(-\frac{8}{7}, 2\right)$

(d) $\left(\frac{4}{7}, 2\right)$

(e) $(44, 8)$

$f(6)=5$

$7x+2=6 \Rightarrow 7x=4 \Rightarrow x=\frac{4}{7}$

$\Rightarrow g(\frac{4}{7}) = 3+f(7(\frac{4}{7})+2)$

$= 3+f(4+2)$

$= 3+f(6)$

$= 3+5$

$\Rightarrow g(\frac{4}{7})=8$

$\Rightarrow$ the point $(\frac{4}{7}, 8)$ is on the graph of $g(x)$

12. Which sequence of transformations will transform the graph of the function f into the graph of the function g?

$$f(x) = \sqrt{x} + 5 \qquad g(x) = \sqrt{x-3} + 6$$

Possibilities:

(a) shift left by 3 then shift down by 1

(b) shift right by 3 then shift up by 1

(c) shift left by 1 then shift down by 3

(d) shift right by 3 then shift down by 1

(e) shift left by 3 then shift up by 1

$f(x-3) = \sqrt{x-3}+5$

$f(x-3)+1 = \sqrt{x-3}+5+1$

$= \sqrt{x-3}+6$

$= g(x)$

$\Rightarrow g(x) = f(x-3)+1$

shift right 3; shift up 1

13. Use algebra to find the inverse of the given one-to-one function.

$$f(x) = (x^7 + 6)^4$$

Possibilities:

(a) $f^{-1}(x) = x^{28} + 6$

(b) $f^{-1}(x) = \sqrt[7]{\sqrt[4]{x} - 6}$

(c) $f^{-1}(x) = (x^4 + 6)^7$

(d) $f^{-1}(x) = \sqrt[4]{\sqrt[7]{x} - 6}$

(e) $f^{-1}(x) = \sqrt[7]{\sqrt[6]{x} - 4}$

$\Rightarrow y = (x^7 + 6)^4$

$\Rightarrow x = (y^7 + 6)^4$

$\Rightarrow \sqrt[4]{x} = y^7 + 6$

$\Rightarrow y^7 = \sqrt[4]{x} - 6$

$\Rightarrow y = \sqrt[7]{\sqrt[4]{x} - 6}$

$\Rightarrow f^{-1}(x) = \sqrt[7]{\sqrt[4]{x} - 6}$

14. Use algebra to find the inverse of the given one-to-one function. $f(x) = \dfrac{6x}{5x+3}$

Possibilities:

(a) $f^{-1}(x) = \dfrac{5x+3}{6x}$

(b) $f^{-1}(x) = \dfrac{3x}{6x+5}$

(c) $f^{-1}(x) = \dfrac{6}{5}x + 3$

(d) $f^{-1}(x) = \dfrac{3x}{6-5x}$

(e) $f^{-1}(x) = \dfrac{6x}{5x-3}$

$y = \dfrac{6x}{5x+3}$

$\Rightarrow x = \dfrac{6y}{5y+3}$

$\Rightarrow (5y+3)x = 6y$

$\Rightarrow 5yx + 3x = 6y$

$\Rightarrow 3x = 6y - 5yx$

$\Rightarrow 3x = y(6-5x)$

$\Rightarrow \boxed{y = \dfrac{3x}{6-5x}}$

15. A weekly census of the tree-frog population in Frog Hollow State Park produces the following results.

Week:	1	2	3	4	5	6
Frogs:	75	225	675	2025	6075	18225

Which exponential growth model most closely matches the observations, if t is the week number?

$\frac{225}{75} = 3$, $\frac{675}{225} = 3$, $\frac{2025}{675} = 3$, $\frac{6075}{2025} = 3$, $\frac{18225}{6075} = 3$

Possibilities:

(a) $3\left(75^{(t/7)}\right)$

(b) $75\left(9^{t}\right)$

(c) $25\left(9^{(t/7)}\right)$

(d) $3\left(75^{t}\right)$

(e) $25\left(3^{t}\right)$

$A(t)$ = size of population at time t (in weeks)

$A(1) = 75 = 3 \cdot 25$
$A(2) = 3 \cdot A(1) = 3^2 \cdot 25$
$A(3) = 3 \cdot A(2) = 3 \cdot 3A(1) = 3^3 \cdot 25$
$A(4) = 3 \cdot A(3) = 3 \cdot 3 \cdot 3A(1) = 3^4 \cdot 25$
$A(5) = 3 \cdot A(4) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot A(1) = 3^5 \cdot 25$
$A(6) = 3 \cdot A(5) = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot A(1) = 3^6 \cdot 25$
$A(t) = 3^t \cdot 25$

16. Determine how much money will be in a savings account if the initial deposit was \$150 and the interest rate is 1.88% compounded continuously for 3 years, 11 months. (Round your answer to the nearest cent.)

$A(t) = Pe^{rt} = 150e^{0.0188 \cdot (3 + \frac{11}{12})} = 161.46$

Possibilities:

(a) \$161.24

(b) \$161.35

(c) \$161.46

(d) \$161.57

(e) \$161.68

17. Translate the given logarithmic statement into an equivalent exponential one.

$\log_6(5x+3) = 17$

The exponent I put on 6 to get $5x+3$

$\Rightarrow 6^{17} = 5x+3$

Possibilities:

(a) $(5x+3)^6 = 17$

(b) $(6)^{5x+3} = 17$

(c) $(17)^6 = 5x+3$

(d) $(5x+3)^{17} = 6$

(e) $(6)^{17} = 5x+3$

18. Write the domain of the function $h(x) = \log(x-5)$ in interval notation.

Possibilities:

(a) $(5, \infty)$

(b) $(-\infty, -5)$

(c) $(-\infty, \infty)$

(d) $(-\infty, 5) \cup (5, \infty)$

(e) $(-\infty, 5]$

$x - 5 > 0 \Rightarrow x > 5$

We cannot put zero or negative numbers into logarithms.

19. Write the given expression as a single logarithm.

$$6\log(x) + \log(5y) - \log(3z)$$

$= \log(x^6) + \log(5y) - \log(3z)$

$= \log(x^6(5y)) - \log(3z)$

$= \log\left(\frac{x^6(5y)}{3z}\right)$

Possibilities:

(a) $\log\left(\frac{x^6y^5}{z^3}\right)$

(b) $\log(6x(5+y) - 3 - z)$

(c) $\log(6x + 5y - 3z)$

(d) $\log\left(\frac{x^6(5y)}{3z}\right)$

(e) $\log(x^6y^5z^3)$

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20. Solve the equation.

$$\log(x-6) + \log(x-4) = \log(7x-48)$$

$\Rightarrow \log((x-6)(x-4)) = \log(7x-48)$

$\Rightarrow (x-6)(x-4) = 7x - 48$

$\Rightarrow x^2 - 10x + 24 - 7x + 48 = 0$

$\Rightarrow x^2 - 17x + 72 = 0$

$\Rightarrow (x-9)(x-8) = 0$

$\Rightarrow x = 9$ or $x = 8$

plugging in both value of x does not produce errors so both values are solutions

Possibilities:

(a) $x = -8$ and $x = -7$

(b) $x = -5$ and $x = -3$

(c) $x = 9$ and $x = 8$

(d) $x = 6$ and $x = 4$

(e) $x = \frac{38}{5}$ only

Formula Sheet:

Compound Interest: If a principal P_0 is invested at an interest rate r for a period of t years, then the amount $P(t)$ of the investment is given by:

$$P(t) = P_0\left(1 + \frac{r}{n}\right)^{nt} \quad \text{(if compounded } n \text{ times per year)}$$

$$P(t) = P_0\, e^{rt} \quad \text{(if compounded continuously).}$$

Change of Base Formula: Let a and b be two positive numbers with $a, b \neq 1$. If $x > 0$, then:

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$