Math 114 Exam 1

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. The wise student will show work for the multiple choice problems as well.

Multiple Choice Questions

1	A	B	C	D	E	**6**	A	B	C	D	E
2	A	B	C	D	E	**7**	A	B	C	D	E
3	A	B	C	D	E	**8**	A	B	C	D	E
4	A	B	C	D	E	**9**	A	B	C	D	E
5	A	B	C	D	E	**10**	A	B	C	D	E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Which trig substitution should be used to find $\int \frac{1}{4+x^2}\,dx$?

   A. $x = 4\tan\theta$

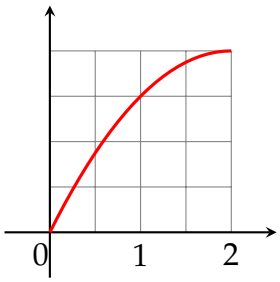
   B. $x = 2\tan\theta$

   C. $x = 2\sin\theta$

   D. $x = 4\sin\theta$

   E. $x = \sin(2\theta)$

2. (5 points) The left endpoint method (L_n), the right endpoint method (R_n), and the Trapezoid method (T_n) are used to estimate $I = \int_0^2 f(x)\,dx$ where the graph of $f(x)$ is as shown. Which of the following is correct for a given n?



   A. L_n overestimates I, T_n underestimates I, and R_n underestimates I

   B. L_n and R_n underestimate I, and T_n overestimates I

   C. L_n and T_n underestimate I, but R_n overestimates I

   D. L_n and T_n overestimate I, and R_n underestimates I

   E. L_n, R_n and T_n all overestimate I

3. (5 points) For what values of p does the improper integral

$$\int_1^\infty \frac{1}{x^{2p}}\,dx$$

   converge?

   A. $p \le 1$

   B. $p \ge 1/2$

   C. $p \le 1/2$

   D. $p > 1/2$

   E. $p < 1$

4. (5 points) The partial fraction decomposition of $\dfrac{1}{x^2+x^4}$ is

   A. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}+\dfrac{D}{x-1}$

   B. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}+\dfrac{D}{(x+1)^2}$

   C. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+1}$

   D. $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{(x+1)^2}$

   E. $\dfrac{Ax+B}{x^2}+\dfrac{C}{(x-1)^2}$

5. (5 points) If $x=\sin(u)$ and $-\pi/2 \le u \le \pi/2$, find $\cot(u)$.

   A. $\sqrt{1-x^2}$

   B. $\sqrt{1-x^2}/x$

   C. $1/\sqrt{1-x^2}$

   D. $x/\sqrt{1-x^2}$

   E. $1/x$

6. (5 points) If we substitute $x = 4\sin u$ with $-\pi/2 \le \theta \le \pi/2$ in the integral

$$\int x\sqrt{16-x^2}\,dx,$$

   we obtain

   A. $\displaystyle\int 64\sin^2(u)\cos(u)\,du$

   B. $\displaystyle\int 16\sin^2(u)\cos(u)\,du$

   C. $\displaystyle\int 16\sin(u)\cos^2(u)\,du$

   D. $\displaystyle\int 64\sin(u)\cos^2(u)\,du$

   E. $\displaystyle\int 64\sin^2(u)\cos^2(u)\,du$

7. (5 points) How large should we take n in the Trapezoid rule in order to approximate $\int_1^2 (1/x)\,dx$ to within 0.0001? Recall that the error E_T made in applying the Trapezoid rule T_n to compute $\int_a^b f(x)\,dx$ obeys the bound

$$E_T \leq \frac{K(b-a)^3}{12n^2}$$

where K is an upper bound for $f''(x)$ on $[a,b]$.

A. $n = 41$ or larger
B. $n = 40$ or less
C. $n = 20$
D. $n = 10$
E. $n = 5$

8. (5 points) Evaluate $\displaystyle\int \frac{5x+1}{(2x+1)(x-1)}\,dx$

A. $\ln|2x+1| + \ln|x-1| + C$
B. $\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C$
C. $\frac{1}{5}\ln|2x+1| + \ln|x-1| + C$
D. $2\ln|2x+1| + \frac{1}{2}\ln|x-1| + C$
E. $\frac{1}{2}\ln|2x+1| + \frac{1}{2}\ln|x-1| + C$

9. Evaluate $\displaystyle\int x\cos x\,dx$

A. $x^2\cos x + x\sin x + C$
B. $x\cos x + \sin x + C$
C. $x\sin x + \cos x + C$
D. $x^2\sin x + C$
E. $x^2\cos x + C$

10. Consider the integral

$$\int_e^\infty \frac{1}{x(\ln x)^2}\,dx.$$

Which of the following statements is correct?

A. The integral is divergent
B. The integral is convergent and its value is 2
C. The integral is convergent and its value is 1
D. The integral is convergent and its value is $1/e$
E. None of these

Free Response Questions

11. (10 points) Compute $\int \frac{10}{(x-1)(x^2+9)}\,dx$

12. (10 points) The following table shows the speedometer reading from a car in 1 minute intervals. Use Simpson's rule to estimate the distance travelled by the car over the 10 minute period. Be careful to make a consistent choice of units and be sure to show your work.

t (min)	0	1	2	3	4	5	6	7	8	9	10
v (mi/h)	40	42	45	49	52	54	56	57	57	55	56

13. (10 points) Compute the definite integral $\int_0^{\pi/2} \sin^2 x \cos^3 x \, dx$.

14. (a) (5 points) Use integration by parts to compute the indefinite integral

$$\int x^2 e^{-x}\, dx$$

(b) (5 points) Determine whether the improper integral

$$\int_0^\infty x^2 e^{-x}\, dx.$$

converges and if so find its value. Recall that $\lim_{x\to\infty} x^2 e^{-x} = \lim_{x\to\infty} xe^{-x} = 0$ by L'Hospital's rule.

15. (10 points) Using trig substitution, evaluate the indefinite integral

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx.$$