Russell Brown **MA114S21**
Assignment Exam01 due 02/20/2021 at 11:59pm EST

Problem 1.
Let a, b be real numbers and consider the integral $\int (ax^2 + b)\cos(x)\,dx$. Using integration by parts will lead to which of the following expressions?

- A. $(ax^2+b)\cos(x)+2a\int x\sin(x)\,dx$
- B. $2ax\cos(x)+2\int(ax^2+b)\sin(x)\,dx$
- C. $(ax^2+b)\sin(x)-2a\int x\sin(x)\,dx$
- D. $(ax^2+b)\cos(x)-2a\int x\cos(x)\,dx$
- E. $2ax\sin(x)-2\int(ax^2+b)\sin(x)\,dx$

Problem 2.
Consider the integral $I = \int_0^4 e^{-x}\,dx$. Let T_n, L_n, and R_n be the approximations to I by the trapezoid rule, the left endpoint rule and the right endpoint rule. Which of the following is true?

- A. $L_n \le T_n \le I \le R_n$
- B. $R_n \le T_n \le I \le L_n$
- C. $L_n \le I \le T_n \le R_n$
- D. $R_n \le I \le T_n \le L_n$
- E. $L_n \le I \le R_n \le T_n$

Problem 3.
What substitution should we make to evaluate $\int \sqrt{4-9x^2}\,dx$?

- A. $x = \frac{3}{2}\sin(u)$
- B. $x = \frac{2}{3}\sin(u)$
- C. $x = 2\sin(u)$
- D. $u = \frac{3}{2}\sin(x)$
- E. $u = \frac{2}{3}\sin(x)$

Problem 4.
Which is the correct form of the partial fraction expansion of $\frac{x^2+7x-11}{(x^2+4x+7)(x^2-1)(x+1)}$?

- A. $\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+7}$
- B. $\frac{A}{x+1}+\frac{Bx+C}{x^2-1}+\frac{Dx+E}{x^2+4x+7}$
- C. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{D}{x^2+4x+7}$
- D. $\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-1}+\frac{Dx+E}{x^2+4x+7}$
- E. $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}+\frac{Dx+E}{x^2+4x+7}$

Problem 5.
Use the decomposition

$$\frac{x^2-x+6}{x^3+3x} = -\frac{x+1}{x^2+3}+\frac{2}{x}$$

to evaluate the integral

$$\int \frac{x^2-x+6}{x^3+3x}\,dx$$

- A. $-\frac{1}{2}\ln(x^2+3)-\frac{1}{\sqrt{3}}\arctan(x/\sqrt{3})+\ln(x^2)+C$
- B. $\frac{1}{\sqrt{3}}\ln(x^2+3)-\frac{1}{\sqrt{2}}\arctan(x/\sqrt{2})+\ln(x^2)+C$
- C. $\ln(x^2+3)+\frac{1}{\sqrt{3}}\arctan(x/\sqrt{2})+\ln(x^2)+C$
- D. $\frac{1}{\sqrt{3}}\ln(x^2+3)-\arctan(x/\sqrt{2})+\ln(x^2)+C$
- E. $\ln(x^2+3)-\arctan(x/\sqrt{3})+\ln(x^2)+C$

Problem 6.
Let $a > 0$ be a fixed number. Evaluate the improper integral $\int_a^\infty x^2e^{-x^3}\,dx$.

- A. ∞
- B. 0
- C. $-\frac{1}{e^{a^3}}$
- D. $\frac{1}{3e^{a^3}}$
- E. e^{a^3}

Problem 7.

If $x = \sin(u)$ and $-\pi/2 < u < \pi/2$, express $\cot(u)$ in terms of x.

- A. $\frac{1}{x}$
- B. $\sqrt{1-x^2}$
- C. $\frac{\sqrt{1-x^2}}{x}$
- D. $\frac{1}{\sqrt{1-x^2}}$
- E. $\frac{x}{\sqrt{1-x^2}}$

Problem 8.

8. **(5 points)** `local/rmb-problems/lim-seq-num.pg`

Consider the sequence $a_n = \frac{5n^2+3n+6}{4n^2+3n-2}$. Find the value of the limit $\lim_{n\to\infty} a_n$.

$\lim_{n\to\infty} a_n =$ __________

Your answer should be correctly rounded to three decimal places, or more accurate. Exact answers are preferred.

This is the free response part of Exam 1. There are 3 questions, each worth 20 points. Please write your solutions in full, clearly indicating each step leading to the final answer. Omitting details will result in a lower grade.

Question 1. (a) Use an appropriate u-substitution to evaluate

$$\int \frac{e^{-1/x}}{x^2}\,dx.$$

(b) Determine whether the improper integral

$$\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$$

converges and if it does converge, determine its value.

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Question 2. Evaluate the integral

$$\int \frac{3x^2 + 5x + 3}{x^3 + x}\, dx.$$

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Question 3. Evaluate

$$\int \frac{\sin^3(x)}{\cos^2(x)}\,dx.$$

(end of exam questions)