Exam 1

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5"X11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1	(A)	(B)	(C)	(D)	(E)	**6**	(A)	(B)	(C)	(D)	(E)
2	(A)	(B)	(C)	(D)	(E)	**7**	(A)	(B)	(C)	(D)	(E)
3	(A)	(B)	(C)	(D)	(E)	**8**	(A)	(B)	(C)	(D)	(E)
4	(A)	(B)	(C)	(D)	(E)	**9**	(A)	(B)	(C)	(D)	(E)
5	(A)	(B)	(C)	(D)	(E)	**10**	(A)	(B)	(C)	(D)	(E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$ and $\tan^2(x) + 1 = \sec^2(x)$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Multiple Choice Questions

1. (5 points) Find $\int x \sin(x)\ dx$.

   A. $\sin(x) + \cos(x) + C$.

   B. $\frac{x^2}{2}(\sin(x) - \cos(x)) + C$.

   C. $\sin(x) - x\cos(x) + C$**.**

   D. $\cos(x^2) + \sin(x) + C$.

   E. $2x \sin(x) + C$.

2. (5 points) If $f(0) = 1$, $f(1) = 1$, $f'(0) = 2$ and $f'(1) = 7$, and $f''(x)$ is continuous, what is $\int_0^1 (x-3)f''(x)\ dx$?

   A. 1

   B. 15

   C. -6

   D. 7

   E. -8

3. (5 points) Find $\int (1+\cos(x))^2\ dx$.

   A. $\frac{3}{2}x + 2\sin(x) + \frac{1}{4}\sin(2x) + C$**.**

   B. $1 + \sin^2(x) + C$.

   C. $\cos(2x) + \frac{3}{4}x + C$.

   D. $\cos(2x) + 2\cos(x) + x + C$.

   E. $\cos(x)^2\sin(2x) + C$.

4. (5 points) Which of the following is equal to the integral

$$\int \frac{x^3}{\sqrt{4-x^2}}\ dx$$

   after making the substitution $x = 2\sin(\theta)$?

   A. $4\int \sec(\theta)\ d\theta$.

   B. $4\int \cos^2(\theta)\ d\theta$.

   C. $8\int \sin(\theta)\cos(\theta)\ d\theta$.

   D. $8\int \sin^3(\theta)\ d\theta$**.**

   E. $2\int \sin^2(\theta)\sec(\theta)\ d\theta$.

5. (5 points) Which of the following is equal to the expression $\tan(\arcsin(\frac{x}{4}))$?

   A. $\frac{x}{4}$.

   B. $\frac{x}{\sqrt{16-x^2}}$**.**

   C. $\frac{1}{4}\sqrt{4+x^2}$.

   D. $\frac{x}{\sqrt{4+x^2}}$.

   E. $\frac{16}{\sqrt{x^2-16}}$.

6. (5 points) Find

$$\int_1^\infty \frac{1}{x^{\frac{3}{2}}}\,dx$$

A. ∞
B. $\frac{3}{2}$
C. 2
D. $\frac{2}{3}$
E. $-\infty$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x-2}{(x+1)(x^2+x+1)(x-1)}?$$

A. $\frac{A}{x+1}+\frac{B}{x-1}+\frac{Cx+D}{x^2+x+1}$
B. $\frac{A}{x+1}+\frac{Bx+C}{x^2+x+1}+\frac{Dx+E}{x-1}$
C. $\frac{A}{x+1}+\frac{Bx+C}{x^2+x+1}$
D. $\frac{A}{x+1}+\frac{Bx+C}{x-1}$
E. $\frac{A}{x+1}+\frac{B}{x^2+x+1}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{x}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

A. $B=\frac{1}{3}$
B. $B=0$
C. $B=\frac{2}{3}$
D. $B=\frac{1}{2}$
E. $B=-\frac{1}{3}$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \leq 3$ on the interval $[5, 7]$. Choose the smallest n so that we can be sure that $E_M = |M_n - \int_5^7 f(x)dx| \leq .000001$, where M_n is the midpoint approximation with n intervals.

   A. $n = 15,000$

   B. $n = 1,000$

   C. $n = 500$

   D. $n = 200$

   E. $n = 100$

10. (5 points) Find the Simpson's rule estimate of $\int_1^7 x^2 \, dx$ for $n = 6$.

   A. $S_6 = 19$

   B. $S_6 = 342$

   C. $S_6 = 115$

   D. $S_6 = 114$

   E. $S_6 = 216$

Free Response Questions

11. (10 points) Compute $\displaystyle\int e^x \cos(x)\, dx$.

Solution: Integration by parts: Let $u = e^x$ so $du = e^x dx$, and $dv = \cos(x)dx$ so $v = \sin(x)$. Then, $\int e^x \cos(x)dx = e^x \sin(x) - \int e^x \sin(x)dx$. Now do a 2nd integration by parts with $u = e^x$ and $dv = \sin(x)$ to get $\int e^x \cos(x)dx = e^x \sin(x) - (-e^x \cos(x) - \int e^x(-\cos(x)dx) = e^x(\sin(x) + \cos(x)) - \int e^x \cos(x)dx$. This gives $\int e^x \cos(x)dx = \frac{1}{2}e^x(\sin(x) + \cos(x)) + C$.

12. (10 points) Compute $\displaystyle\int \sqrt{9+x^2}\, dx$. You must simplify your answer.

Solution: Use the trig substitution $x = 3\tan\theta$ so $\sqrt{9+x^2} = 3\sqrt{1+\tan^2\theta} = 3\sec\theta$ and $dx = 3\sec^2\theta d\theta$. This results in the integral $9\int \sec^3\theta d\theta$.

Now we do integration by parts with $u = \sec\theta$ and $dv = \sec^2\theta d\theta$, giving $du = \sec\theta\tan\theta d\theta$ and $v = \tan\theta$. This gives $9\int \sec^3\theta d\theta = 9\sec\theta\tan\theta - 9\int \tan^2\theta\sec\theta d\theta$.

Now we use the trig identity $\tan^2\theta = \sec^2\theta - 1$ to obtain $9\int\sec^3\theta d\theta = 9\sec\theta\tan\theta - 9\int(\sec^2\theta - 1)\sec\theta d\theta = 9\sec\theta\tan\theta + 9\int\sec\theta d\theta - 9\int\sec^2\theta d\theta = 9\sec\theta\tan\theta + 9\ln(|\sec\theta+\tan\theta|) - 9\int\sec^3\theta d\theta$. As a consequence we get
$9\int\sec^3\theta d\theta = \frac{9}{2}(\sec\theta\tan\theta + \ln(|\sec\theta+\tan\theta|)) + C$.

Now we substitute $\theta = \arctan(\frac{x}{3})$ to get

$\int\sqrt{9+x^2}dx =$

$\frac{9}{2}(\sec(\arctan(\frac{x}{3}))\tan(\arctan(\frac{x}{3})) + \ln(|\sec(\arctan(\frac{x}{3})) + \tan(\arctan(\frac{x}{3}))|)) + C =$

$\frac{9}{2}(\sec(\arctan(\frac{x}{3}))\frac{x}{3}) + \ln(|\sec(\arctan(\frac{x}{3})) + \frac{x}{3})|)) + C$

We compute $\sec(\arctan(\frac{x}{3})) = \frac{1}{3}\sqrt{9+x^2}$ to get:

$\int\sqrt{9+x^2}dx = \frac{9}{2}(\frac{x}{9}\sqrt{9+x^2}) + \ln(|\frac{1}{3}\sqrt{9+x^2} + \frac{x}{3})|)) + C$

13. (10 points) Compute $\displaystyle\int_1^\infty xe^{x^2+1}\,dx$. **Justify your answer by showing your work.**

Solution: The improper integral is defined by $\lim_{t\to\infty}\int_1^t xe^{x^2+1}dx$.

Letting $u = x^2+1$ we get $du = 2xdx$, so the indefinite integral is $\int xe^{x^2+1}dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^{x^2+1} + C$.

The definite integral is $\int_1^t xe^{x^2+1}dx = [\frac{1}{2}e^{x^2+1}]_1^t = \frac{1}{2}[e^{t^2+1} - e]$.

Taking the limit, we obtain $\lim_{t\to\infty}\frac{1}{2}[e^{t^2+1} - e] = \lim_{t\to\infty}\frac{1}{2}e^{t^2+1} - \frac{e}{2} = \infty$.

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{1}{(x-3)(x^2-2x+1)}\,dx.$$

Solution:

The partial fraction is:

$$\frac{1}{(x-3)(x^2-2x+1)} = \frac{1}{(x-3)(x-1)^2} = \frac{A}{x-3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

This results in:

$$1 = A(x-1)^2 + B(x-3)(x-1) + C(x-3)$$

Solving, we get $A = \frac{1}{4}, B = -\frac{1}{4}, C = -\frac{1}{2}$. The integral becomes

$$\frac{1}{4}\int \frac{dx}{x-3} - \frac{1}{4}\int \frac{dx}{x-1} - \frac{1}{2}\int \frac{dx}{(x-1)^2}$$

leading to

$$\frac{1}{4}\ln(x-3) - \frac{1}{4}\ln(x-1) + \frac{1}{2}(\frac{1}{x-1}) + C$$

15. (a) (5 points) Use the midpoint rule to estimate the integral

$$\int_1^7 \frac{1}{x^3}\,dx$$

Use six intervals (ie find M_6).

Solution: Midpoint rule for $f(x) = \frac{1}{x^3}$ and $n = 6$, with $b - a = 7 - 1 = 6$, so $\Delta = 1$.

Endpoints are $1, 2, 3, 4, 5, 6, 7$. The midpoints are $\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}$.

$$M_6 = \frac{2^3}{3^3} + \frac{2^3}{5^3} + \frac{2^3}{7^3} + \frac{2^3}{9^3} + \frac{2^3}{11^3} + \frac{2^3}{13^3} = .404246$$

(b) (5 points) Use the trapezoid rule to estimate the integral

$$\int_1^7 \frac{1}{x^3}dx$$

Use six intervals (ie find T_6).

Solution: From above, the endpoints are $1, 2, 3, 4, 5, 6, 7$, so

$$T_6 = \frac{1}{2}[\frac{1}{1^3} + 2\frac{1}{2^3} + 2\frac{1}{3^3} + 2\frac{1}{4^3} + 2\frac{1}{5^3} + 2\frac{1}{6^3} + \frac{1}{7^3}] = .691749$$