MA 114 - Calculus II
PRACTICE
SECOND MIDTERM

Spring 2004
03/09/2004

Name: ______________________ Sec.: ____

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	D. Watson	MWF 8:00-8:50, CP 222	TR 8:00-9:15, CB 347
002	A. Corso	D. Watson	MWF 8:00-8:50, CP 222	TR 12:30-1:45, CP 155
003	A. Corso	S. Petrovic	MWF 8:00-8:50, CP 222	TR 3:30-4:45, CB 347

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer
  (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		54
2.		10
3.		15
4.		15
5.		10
Bonus.		5
TOTAL	out of 100 pts	109

1. Evaluate the following integrals. Each problem is worth 7 points.

(a) $\displaystyle\int \sin^3 x \cos^3 x \, dx =$ ______________________________.

(b) $\displaystyle\int \frac{x}{x^2+4x+5} \, dx =$ ______________________________.

(c) $\displaystyle\int \sqrt{x} \ln(5x) \, dx =$ ______________________________.

(d) $\displaystyle\int \frac{1+\sin x}{\cos^2 x} \, dx =$ ______________________________.

pts: /28

1.(cont.d)

(e) (7 pts) $\displaystyle\int \frac{1}{(x^2+1)^{\frac{3}{2}}}\,dx =$ ______________________________.

(f) (9 pts) For each of the following functions write out the form of the partial fractions decomposition. DO NOT solve for the coefficients.

$\displaystyle\frac{x}{(x+1)(x+4)} =$ ______________________________.

$\displaystyle\frac{x^2+1}{x^4+x^3+2x^2} =$ ______________________________.

$\displaystyle\frac{x}{x^4+2x^2+1} =$ ______________________________.

(g) Find the partial fraction decomposition of the function $f(x)$ (5 pts) and then evaluate the corresponding integral (5 pts):

$\displaystyle f(x) = \frac{1}{x^4+x^2} =$ ______________________________,

$\displaystyle\int \frac{1}{x^4+x^2}\,dx =$ ______________________________.

pts: /26

The **trapezoid rule** T_n and **Simpson's rule** S_n for approximating the integral $\displaystyle\int_a^b f(x)dx$ are:

$$T_n = \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + \cdots\cdots + 2f(x_{n-1}) + f(x_n)\right),$$

$$S_n = \frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots\cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right),$$

where $\Delta x = b - a/n$, $x_0 = a$, $x_i = x_0 + i\Delta x$ for $i = 1, \ldots, n$, and n is even in Simpson's rule.
The **error** in the trapezoid rule, E_T, and in Simpson's rule, E_S, satisfy

$$|E_T| \leq \frac{K_2(b-a)^3}{12n^2} \qquad \text{and} \qquad |E_S| \leq \frac{K_4(b-a)^5}{180n^4}$$

where K_j is a number so that the jth derivative satisfies $|f^{(j)}(x)| \leq K_j$ for all x with $a \leq x \leq b$.

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2. Consider the integral $\displaystyle\int_0^2 e^{-x^2}\,dx$.

(a) Use the trapezoid rule with $n = 5$ to estimate the above integral. Round your answer to 3 decimal places.

(b) Use Simpson's rule with $n = 4$ to estimate the above integral. Round your answer to 3 decimal places.

pts: /10

3. (a) (5 pts) State the Comparison Theorem for integrals.

(b) (5 pts) Use the Comparison Theorem to determine whether the following integral converge or diverge

$$\int_0^\infty \frac{\sin^2(x)}{1+x^2}\,dx.$$

(c) (5 pts) Use the Comparison Theorem to determine whether the following integral converge or diverge

$$\int_1^\infty \frac{2+e^{-x}}{1+x}\,dx.$$

pts: /15

4. A model for a growth function for a limited population is given by the *Gompertz function* which is a solution of the differential equation

$$\frac{dy}{dt} = c \ln\left(\frac{M}{y}\right) y$$

where c is a constant and M is the maximum size of the population.

(a) (12 pts) Solve the differential equation.

$y(t) =$ ______________________________

(b) (3 pts) Compute $\lim_{t\to\infty} y(t) =$ __________.

pts: /15

5. Find the length of the curve

$$y = \int_0^x \sqrt{3t^4 - 1}\,dt$$

from $x = -2$ to $x = -1$.

pts: /10

Bonus. Consider the integral $\displaystyle\int_1^3 e^{-3x}\,dx$.

(a) Find n so that the error in approximating the above integral by the trapezoid rule T_n is less than 10^{-4}.

(b) Find n so that the error in approximating the above integral by Simpson's rule S_n is less than 10^{-4}.

pts: /5