

Exam 3

Name: _____ Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 A B C D E6 A B C D E2 A B C D E7 A B C D E3 A B C D E8 A B C D E4 A B C D E9 A B C D E5 A B C D E10 A B C D E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig identities

- $\sin^2(x) + \cos^2(x) = 1$,
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$
- $\frac{d}{dx} \cot(x) = -\csc^2(x)$

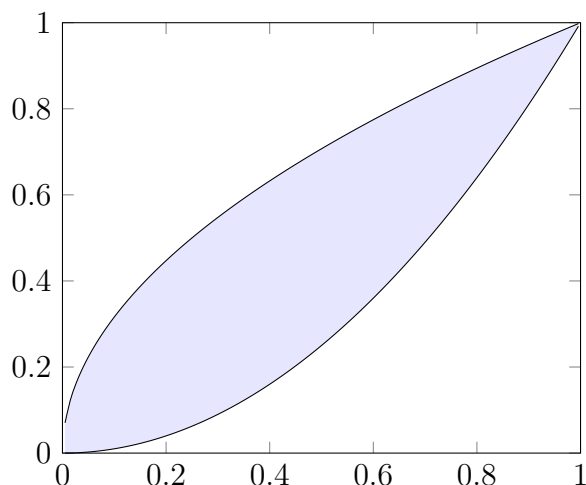
Multiple Choice Questions

1. (5 points) What is the average value of the function $\csc^2(x)$ on the interval $[\frac{\pi}{4}, \frac{3\pi}{4}]$?
 - A. 4π
 - B. $\frac{\pi}{4}$
 - C. $\frac{\pi}{2}$
 - D. $\frac{4}{\pi}$
 - E. $\frac{2}{\pi}$

2. (5 points) Which of the following is a parametrization of the circle of radius 1 centered at $(0, 0)$.
 - A. $x(t) = \cos(3t), y(t) = 1 + \sin(3t)$
 - B. $x(t) = 1 + \sin(t), y(t) = -1 + \cos(t)$
 - C. $x(t) = \sin(5t), y(t) = \cos(5t)$
 - D. $x(t) = 3\cos(t), y(t) = 2\sin(t)$
 - E. $x(t) = 2\sqrt{1-t}, y(t) = 2\sqrt{1+t}$

3. (5 points) Three masses are located in the plane: 3 grams at $(-5, 1)$, 2 grams at $(1, 1)$, and 6 grams at $(-6, 0)$. Find the center of mass of this system.
 - A. $(-4, \frac{5}{12})$
 - B. $(\frac{-49}{10}, \frac{1}{2})$
 - C. $(-4, \frac{1}{2})$
 - D. $(\frac{-4911}{11}, \frac{5}{11})$
 - E. $(\frac{-1}{4}, 2)$

4. (5 points) The region bounded by $y = \sqrt{x}$ and $y = x^2$ is shown below.



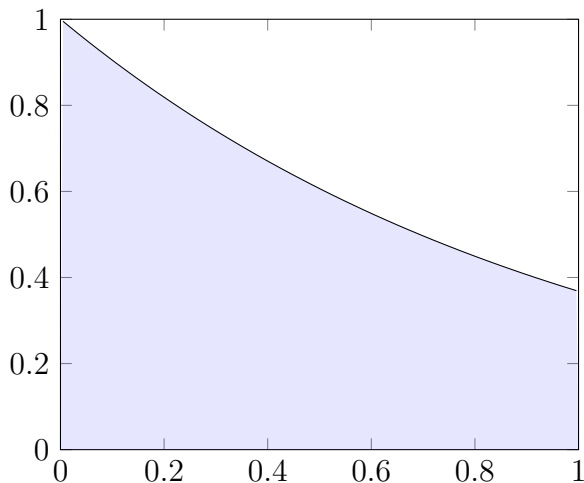
Consider the solid obtained by rotating this region around the x -axis. Using the washer method, which integral will compute the volume of this solid?

- A. $\int_0^{\sqrt{2}} 2\pi (\sqrt{x} - x^2)^2 dx.$
- B. $\int_0^1 \pi (x - x^4) dx.$
- C. $\int_0^1 \pi (x^2 - x^4) dx.$
- D. $\int_0^1 \pi (\sqrt{x} - x^2)^2 dx.$
- E. $\int_0^1 \pi (\sqrt{x} - x^2) dx.$

5. (5 points) Which integral below computes the length of the arc parametrized by (x, x^2) where $0 \leq x \leq 1$?

- A. $\int_0^1 1 + 4x^2 dx.$
- B. $\int_0^1 \sqrt{1 + 4x^2} dx.$
- C. $\int_0^1 \sqrt{1 + \frac{x^2}{4}} dx.$
- D. $\int_0^1 \sqrt{1 + 4x} dx.$
- E. $\int_0^1 2\pi x^2 \sqrt{1 + 4x^2} dx.$

6. (5 points) The region bounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$ and $x = 1$ is shown below.



Consider the solid obtained by rotating this region about the y -axis. Using the shell method, which integral will compute the volume of this solid?

- A. $\int_0^1 2\pi x e^{-x} dx.$
- B. $\int_0^1 2\pi e^{-x} dx.$
- C. $\int_0^1 2\pi x (e^{-x})^2 dx.$
- D. $\int_0^1 2\pi x^2 e^{-x} dx.$
- E. $\int_0^e 2\pi x^2 e^{-x} dx.$

7. (5 points) The line $y = 2x + 1$ for $1 \leq x \leq 4$ is rotated about the x -axis. What is the area of the resulting surface?

- A. 36π
- B. $18\sqrt{3}\pi$
- C. $36\sqrt{3}\pi$
- D. $\sqrt{3}\pi$
- E. $36\sqrt{5}\pi$

8. (5 points) Find the slope of the tangent line to the curve parametrized by $x(t) = t^2 + 1, y(t) = t^3$ at the point $(2, 1)$.

- A. $\frac{3}{2}$
- B. 2
- C. $\frac{2}{5}$
- D. $\frac{4}{5}$
- E. $-\frac{3}{2}$

9. (5 points) Find the length of the curve parametrized by $x(t) = e^t \cos(t), y(t) = e^t \sin(t), 0 \leq t \leq \pi$.

- A. $\pi - \sqrt{2}$
- B. $\sqrt{2}(e^\pi - 1)$
- C. $\sqrt{2} - 2$
- D. $e^\pi - e$
- E. $\sqrt{2}\pi$

10. (5 points) The curve $y = 1 + e^x$ for $0 \leq x \leq 2$ is rotated about the y -axis, producing a surface. Which of the following integrals calculates its surface area?

- A. $\int_2^{1+e^2} 2\pi \ln(y-1) \sqrt{1 + \left(\frac{1}{y-1}\right)^2} dy$
- B. $\int_1^{1+e^2} 2\pi y \sqrt{1 + \left(\frac{1}{y-1}\right)^2} dy$
- C. $\int_0^2 2\pi(1 + e^x) \sqrt{1 + e^{2x}} dx$
- D. $\int_0^2 2\pi x \sqrt{1 + (1 + e^x)^2} dx$
- E. $\int_0^2 2\pi(1 + e^x) \sqrt{1 + x^2} dx$

Free Response Questions

11. The *cycloid* for the circle of radius 1 is the curve parametrized by the functions

$$x(\theta) = \theta - \sin(\theta),$$

$$y(\theta) = 1 - \cos(\theta).$$

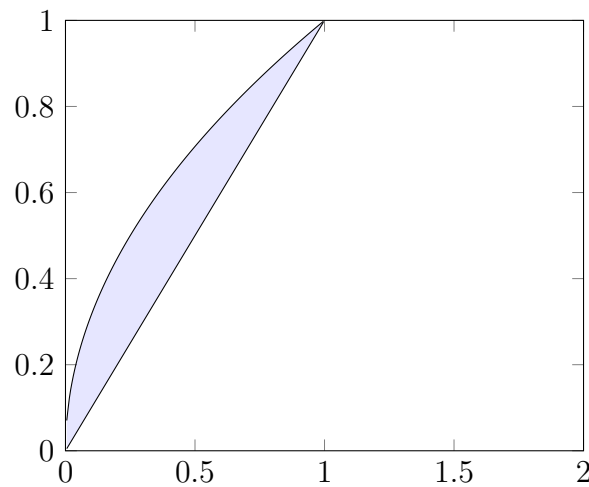
- (a) (3 points) Find the coordinates of the point P on the cycloid given by $\theta = \frac{\pi}{4}$.
- (b) (3 points) Find the slope of the tangent line to the cycloid at P .
- (c) (4 points) Find the arc length of the piece of the cycloid parametrized by $0 \leq \theta \leq 2\pi$.

12. Let S be the region in the plane bounded by the parabola $y = x - x^2$ for $0 \leq x \leq 1$ and the x -axis.

(a) (7 points) Find the total mass M and the moments M_y and M_x for S . Assume that S has uniform density $\rho = 1$.

(b) (3 points) Find the center of mass of S .

13. The region between the curves $y = \sqrt{x}$ and $y = x$ is shown below.



Let V be the solid obtained by rotating this region around the line $x = 2$.

- (a) (3 points) Set up but do not evaluate the integral that computes the volume of V using the disk/washer method.
- (b) (3 points) Set up but do not evaluate the integral that computes the volume of V using the cylindrical shells method.
- (c) (4 points) Find the volume of V by evaluating one of the integrals you found in parts (a) or (b).

14. Let S be the surface obtained by revolving the arc L formed by the graph of $\sqrt{x+1}$ with $0 \leq x \leq 5$ around the x axis.

(a) (2 points) Set up but do not evaluate an integral which computes the arc length of L .

(b) (3 points) Set up but do not evaluate an integral which computes the surface area of S .

(c) (5 points) Find the surface area of S .

15. Let P be the pyramid of height 10 and rectangular base of length 15 and width 20.

(a) (5 points) Find a function giving the area of the cross-section of P at height z .

(b) (3 points) Set up an integral which computes the volume of P .

(c) (2 points) Find the volume of P .