# *Exam 3*

Name: ______________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

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Multiple Choice Questions

**1** (A) (B) (C) (D) (E) **6** (A) (B) (C) (D) (E)

**2** (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

**3** (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

**4** (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

**5** (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

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| Multiple Choice | 11 | 12 | 13 | 14 | 15 | Total Score |
|---|---|---|---|---|---|---|
| 50 | 10 | 10 | 10 | 10 | 10 | 100 |
| | | | | | | |

1. (5 points) Find the average value of $f(x) = e^{2x}$ on the interval $[0, 5]$.

   A. $e^{10}$

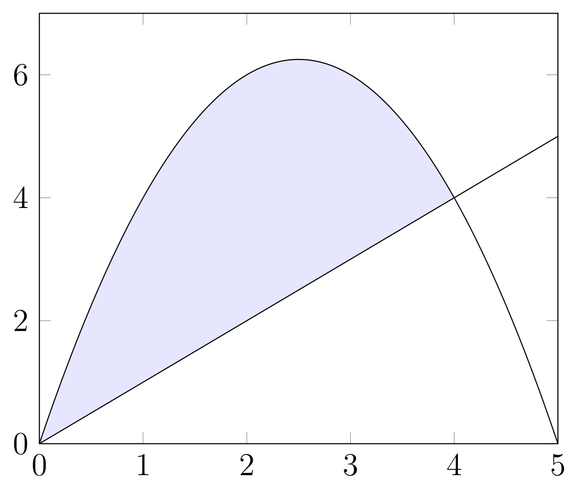
   B. $\frac{2}{5}(e^{10} - 1)$

   **C.** $\frac{1}{10}(e^{10} - 1)$

   D. $\frac{1}{5}(e^{10} - 1)$

   E. $\frac{5}{2}(e^{10} - 1)$

2. (5 points) The region $R$ bounded by $y = 5x - x^2$ and $y = x$ is shown below.




Consider the solid obtained by rotating $R$ about the vertical line $x = 6$. Which integral computes the volume of this solid using the **shell method**?

   A. $\displaystyle\int_0^4 2\pi(x+6)(x^2 - 4x)dx$

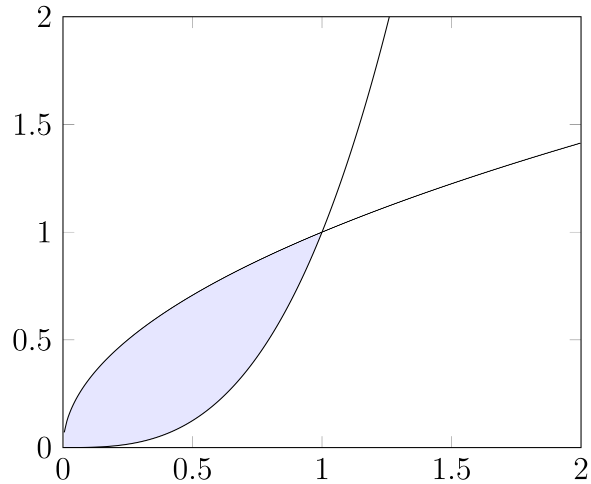
   **B.** $\displaystyle\int_0^4 2\pi(6-x)(4x - x^2)dx$

   C. $\displaystyle\int_0^4 2\pi(5x - x^2 - x - 6)dx$

   D. $\displaystyle\int_0^4 2\pi\left((5x - x^2)^2 - x^2\right)dx$

   E. $\displaystyle\int_0^4 2\pi\left((5x - x^2 - 6)^2 - (x-6)^2\right)dx$

3. (5 points) The region $R$ bounded by the curves $y = x^3$ and $y = \sqrt{x}$ is shown below.




Consider the solid obtained by rotating $R$ about the $\boldsymbol{y}$**−axis**. Which integral computes the volume of this solid using the **disk/washer** method?

**A.** $\int_0^1 \pi((\sqrt[3]{y})^2 - (y^2)^2)dy$

B. $\int_0^1 \pi y((y^3)^2 - (\sqrt{y})^2)dy$

C. $\int_0^1 \pi(y^2 - (\sqrt[3]{y})^2))dy$

D. $\int_0^1 2\pi y(\sqrt[3]{y} - y^2)dy$

E. $\int_0^1 \pi((1 - \sqrt[3]{y})^2 - (1 - y^2)^2)dy$

4. (5 points) Three masses are located in the plane: 2 grams at $(7, 4)$, 3 grams at $(-2, 6)$, and 5 grams at $(1, 8)$. Find the center of mass of this system.

A. $(0.6, 1.8)$

**B.** $(1.3, 6.6)$

C. $(3.1, 7)$

D. $(2, 6)$

E. $(66, 13)$

5. (5 points) Which integral computes the length of the curve $y = f(x)$ where $f(x) = x^5$ and $0 \leq x \leq 2$?

   A. $\int_0^2 \sqrt{1 + 5x^4}dx$

   B. $\int_0^2 \pi(x^{10})dx$

   C. $\frac{1}{2}\int_0^2 x^5 dx$

   D. $\int_0^2 \sqrt{1 + x^{10}}dx$

   **E.** $\int_0^2 \sqrt{1 + 25x^8}dx$

6. (5 points) The line $y = 3x + 1$ for $1 \leq x \leq 5$ is rotated about the $x$-axis. What is the **surface area** of the resulting surface?

   A. $24\pi\sqrt{10}$

   B. $36\pi\sqrt{10}$

   C. $40\pi\sqrt{10}$

   D. $72\pi\sqrt{10}$

   **E.** $80\pi\sqrt{10}$

7. (5 points) The curve $y = 1 + 4x^2$ from $x = 0$ to $x = 2$ is rotated about the $\boldsymbol{y}$**−axis**. Which integral computes the **surface area** of the resulting surface?

   A. $\int_0^2 \sqrt{1 + 64x^2}dx$

   B. $\int_0^2 \pi(1 + 64x^2)^2 dx$

   C. $\int_0^2 2\pi x(1 + 4x^2)dx$

   **D.** $\int_0^2 2\pi x\sqrt{1 + 64x^2}dx$

   E. $\int_0^2 2\pi(1 + 4x^2)\sqrt{1 + 64x^2}dx$

8. (5 points) Which of the following is a parametrization of a curve defined by the equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1?$$

   A. $x(t) = a\cos(t), \quad y(t) = b\sin(t)$

   B. $x(t) = a\sin(t), \quad y(t) = b\cos(t)$

   C. $x(t) = a\cos(2t), \quad y(t) = b\sin(2t)$

   **D. all of the above**

   E. none of the above

9. (5 points) Eliminate the parameter $t$ to find a Cartesian equation satisfied by the curve parametrized by $x(t) = 3t + 7$, $y(t) = 2t - 1$.

   A. $2x + 3y = 6$

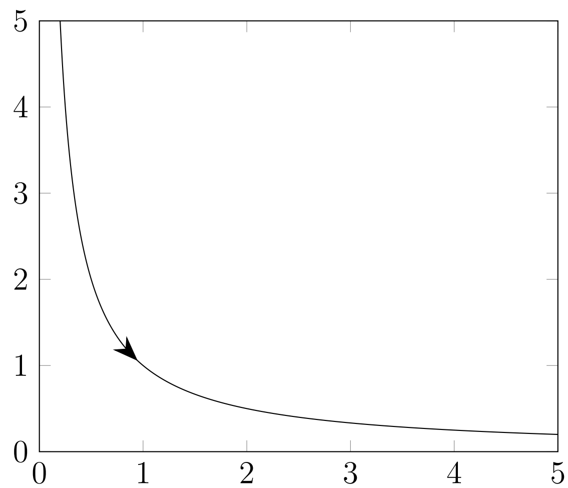
   B. $y = 6x + 13$

   **C.** $y = \frac{2}{3}x - \frac{17}{3}$

   D. $y = 5x + 6$

   E. $y = \frac{2}{3}x - \frac{1}{7}$

10. (5 points) The graph below shows the plot of a parametric curve. Which parametrization is correct?




   A. $x(t) = t^2, \quad y(t) = t^4$

   B. $x(t) = 1/t, \quad y(t) = 1/t, \quad t > 0$

   C. $x(t) = \cos(t), \quad y(t) = \sin(t)$

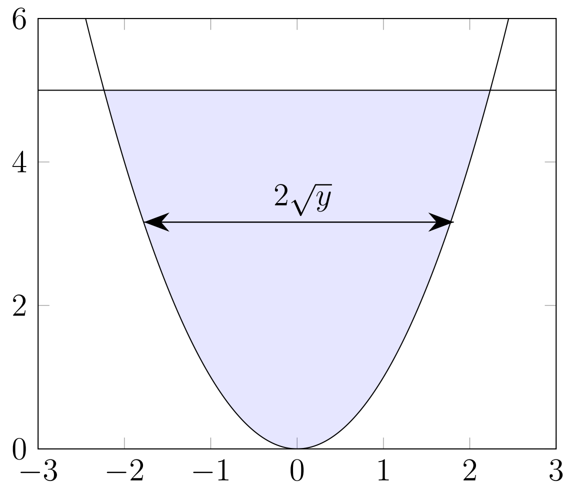
   D. $x(t) = \ln(t), \quad y(t) = \sqrt{t}, \quad t \geq 1$

   **E.** $x(t) = e^t, \quad y(t) = e^{-t}$

Free Response Questions

11. The base of a solid is the region enclosed by $y = x^2$ and $y = 5$. Cross-sections perpendicular to the $y$-axis are squares. Set up an integral which computes the volume of this solid, and then find the volume.

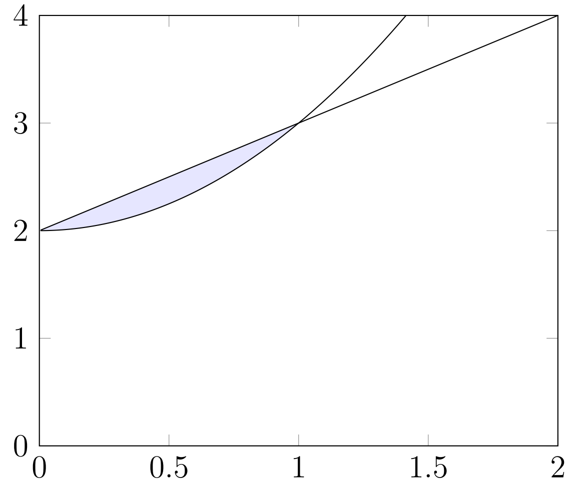
**Solution:**




The base of each square is $2\sqrt{y}$, hence the area of each square is $4y$. Thus, the volume is calculated as follows

$$V = \int_0^5 A(y)\ dy = \int_0^5 4y\ dy = 2y^2\Big|_0^5 = 2(25) - 0 = 50$$

12. The region between $y = x^2 + 2$ and $y = x + 2$ is shown below. Let $V$ be obtained by rotating this region about the horizontal line $y = -1$.




(a) (4 points) Set up but do not evaluate the integral that computes the volume of $V$ using the **disk/washer** method.

**Solution:**

$$V = \int_0^1 \pi \left((x+2+1)^2 - (x^2+2+1)^2\right) dx = \int_0^1 \pi \left((x+3)^2 - (x^2+3)^2\right) dx$$

(b) (6 points) Set up but do not evaluate the integral that computes the volume of $V$ using the **shell** method.

**Solution:**

$$V = \int_2^3 2\pi(y+1)\left(\sqrt{y-2} - (y-2)\right) dy$$

13. Let $S$ be the region in the first quadrant under $y = 8 - x^3$. Assume $S$ has uniform density $\rho = 1$.

(a) (8 points) Find the total mass $M$ and the moments $M_x$ and $M_y$ for $S$. Show all steps clearly. Clearly label each answer.

**Solution:**

$$M = (denity)(area) = 1 \cdot \int_0^2 (8 - x^3)dx = 8x - \frac{x^4}{4}\Big|_0^2 = 16 - \frac{16}{4} = 12$$

$$M_y = \rho \int_a^b xf(x)dx = 1 \cdot \int_0^2 x(8 - x^3)dx = \int_0^2 8x - x^4\ dx$$
$$= 4x^2 - \frac{x^5}{5}\Big|_0^2 = 16 - \frac{32}{5} = \frac{48}{5}$$

$$M_x = \frac{\rho}{2}\int_a^b f(x)^2dx = \frac{1}{2}\int_0^2 (8 - x^3)^2dx = \frac{1}{2}\int_0^2 64 - 16x^3 + x^6\ dx$$
$$= \frac{1}{2}(64x - 4x^4 + \frac{x^7}{7})\Big|_0^2 = \frac{1}{2}(128 - 64 + \frac{128}{7}) = \frac{288}{7}$$

(b) (2 points) Find the center of mass of $S$.

**Solution:** $\bar{x} = \frac{M_y}{M} = \frac{48/5}{12} = \frac{4}{5}$
$\bar{y} = \frac{M_x}{M} = \frac{288/7}{12} = \frac{24}{7}$
The center of mass is $(\bar{x}, \bar{y}) = \left(\frac{4}{5}, \frac{24}{7}\right)$

14. Let $C$ be the curve defined by the graph of $f(x) = e^{3x}$ for $1 \le x \le 7$.

(a) (5 points) Set up but do not evaluate an integral which computes the length of $C$.

**Solution:**

$$L = \int_a^b \sqrt{1 + (f'(x))^2}dx = \int_1^7 \sqrt{1 + (3e^{3x})^2}dx = \int_1^7 \sqrt{1 + 9e^{6x}}dx$$

(b) (5 points) Set up but do not evaluate an integral which computes the surface area when $C$ is rotated about the $x$-axis.

**Solution:**

$$SA = \int_a^b 2\pi r L = \int_1^7 2\pi(e^{3x})\sqrt{1 + (3e^{3x})^2}dx$$

15. (a) (5 points) Find the average value of $f(x) = 2 + 3\sin(x)$ on $[\pi/2, \pi]$. Show all steps clearly.

**Solution:**

$$\begin{aligned}\frac{1}{\pi - \pi/2}\int_{\pi/2}^{\pi} 2 + 3\sin(x)\ dx &= \frac{2}{\pi}(2x - 3\cos(x))\Big|_{\pi/2}^{\pi} \\ &= \frac{2}{\pi}((2\pi - 3\cos(\pi)) - (\pi - 3\cos(\pi/2))) \\ &= \frac{2}{\pi}(2\pi + 3 - \pi + 0) \\ &= \frac{2}{\pi}(\pi + 3)\end{aligned}$$

(b) (5 points) Find a Cartesian equation satisfied by the curve $C$ parametrized by $x(t) = t + 1$, $y(t) = t^2$, $t > 0$; and sketch the graph of $C$.

**Solution:** Since $x = t + 1$ then $t = x - 1$, so $y = (x - 1)^2$. Since $t > 0$ then $x > 1$. Then the graph of the curve is the parabola $y = (x - 1)^2$ for $x > 1$.