MA 114 Calculus II — Exam 3

NAME: SECTION:
LAST FOUR DIGITS OF STUDENT ID:

NOTES:

1. This is a closed book exam. There are six (6) problems on ten (10) pages (including this cover page). Check and be sure that you have a complete exam.

2. You may use a graphing calculator that does not have symbolic manipulation capabilities. Any device capable of electronic communication (cell phone, pager, etc.) must be turned off and out of sight during the exam.

3. Each question is followed by space to write your answer. Please write your solutions neatly in the space below the question. Please erase or mark out any work that you do not want graded. If you need more space, use the backs of the exam pages.

4. Unless specified otherwise, **show your work**; answers without any justification will receive no credit.

Problem	Score	Points
1		18
2		20
3		16
4		20
5		10
6		16
Total		100

1. (a) (6 pts) Write out the form of the partial fraction decomposition of the function $\dfrac{2x-3}{(x-1)(x^2+x+1)^2}$. Don't determine the numerical values of the coefficients.

(b) (12 pts) Use the method of partial fractions to evaluate the integral

$$\int_0^1 \frac{2}{(x+1)(x^2+1)}dx.$$

2. Determine whether each improper integral below is convergent or divergent. If it is convergent, then evaluate it.

(a) (10 pts) $\int_1^\infty \frac{dx}{x(x+1)}$.

(b) (10 pts) $\int_1^2 \frac{x}{\sqrt{x-1}} dx$.

3. (16 pts) Use the integral test to show that the infinite series $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$ is convergent. You need to verify that your choice of f satisfies the hypotheses of this test.

4. (a) The following table gives the values of a function W at the given points in the interval $[0, H]$:

x	0	$H/4$	$H/2$	$3H/4$	H
$W(x)$	0.093	0.067	0.082	0.030	0.009

Let $I = \int_0^H W(x)dx$. In each part below, you may leave your answer as an expression that involves sums and products of numbers or decimals.

(i) (5 pts) Use the Trapezoidal Rule with $n = 4$ to obtain an approximate value of I expressed in terms of H.

(ii) (5 pts) Use Simpson's Rule with $n = 4$ to obtain an approximate value of I expressed in terms of H.

(iii) (2 pts) Use the approximate value of I obtained either in (i) or in (ii) to get an estimate for the average value (or integral average) of W on $[0, H]$.

4. (b) (8 pts) How large should we take n in order to guarantee that the approximation by Simpson's Rule for $\int_1^2 \ln x \, dx$ is accurate to within 10^{-4}? Recall that the error bound for Simpson's rule for $\int_a^b f(x)dx$ is given by

$$|E_S(n)| \leq \frac{K(b-a)^5}{180n^4}, \qquad \text{with} \quad K = \max\Big\{\Big|f^{(4)}(x)\Big| : a \leq x \leq b\Big\}.$$

5. (10 pts) Set up, but do not evaluate, an integral for the length of the curve

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad \text{where } \; a > 0, \;\; b > 0.$$

6. A curve $\mathcal{C}$ is defined by the parametric equations

$$x = t^3 - 3t, \qquad y = 3t^2 - 9.$$

(a) (10 pts) Find the Cartesian coordinates of the points on $\mathcal{C}$ where (i) the tangent is horizontal and (ii) the tangent is vertical.

6. (b) (6 pts) Determine the values of t for which (i) the curve is concave upward and (ii) the curve is concave downward.