Calculus II   Russell Brown

Final 3   17 December 2003

Answer all of the following questions. Additional sheets are available if
necessary. No books or notes may be used. You may use a calculator. You
may not use a calculator which has symbolic manipulation capabilities.
When answering these questions, please be sure to 1) check answers when
possible, 2) clearly indicate your answer and the reasoning used to arrive at
that answer *(unsupported answers may not receive credit)*.
Each question is followed by space to write your answer. Please lay out
your solutions neatly in the space below the question. You are not expected
to write each solution next to the statement of the question.
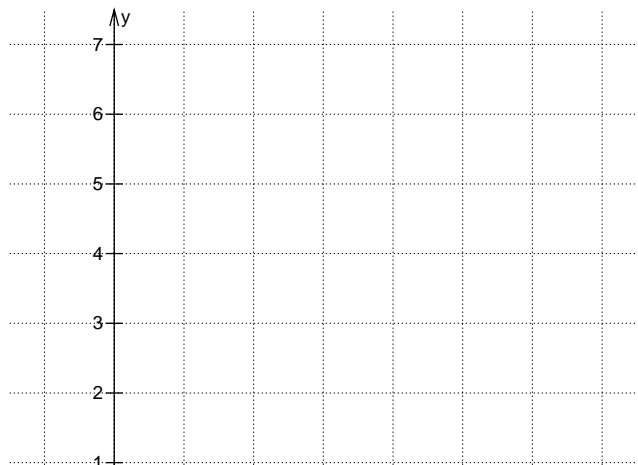

Name _____

Section _____

Question	Score	Total
1		12
2		8
3		18
4		10
5		10
6		5
7		20
8		10
9		12
min(score,100)		100

1. Let $f(x) = \cos x$ for $\pi \le x \le 2\pi$.

   (a) Sketch the graph of f.

   (b) Sketch the graph of f^{-1}.

   (c) Find the the derivative of f^{-1}, $f^{-1\prime}$. Simplify your answer.

2. Suppose that a tank holds 500 liters of water initially. Brine flows in at a rate of 4 liters/minute. The brine has a concentration of 5 grams/liter. The tank is well-stirred and brine flows out a rate of 4 liters/minute.

   (a) Write a differential equation for $M(t)$, the mass of salt in the tank after t minutes.

   (b) Solve the differential equation you wrote in part a) and find the function $M(t)$ for all $t \geq 0$.

   (c) Find the mass of salt in the tank after 15 minutes.

   (d) Find $\lim_{t \to \infty} M(t)$.

3. Determine if the following series are absolutely convergent, conditional convergent, or divergent. Briefly justify each answer.

   (a) $\sum_{n=1}^{\infty} \frac{n}{1+n^2}$.

   (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+1}$.

   (c) $\sum_{n=1}^{\infty} \frac{n^2}{4^n}$.

   (d) $\sum_{n=1}^{\infty} \frac{\sin n}{n^3}$.

4. Compute the following integrals.

   (a) $\int_0^{1/2} \sqrt{1-x^2}\, dx$.

   (b) $\int \frac{x-1}{x^2+4}\, dx$.

   (c) $\int_2^3 \frac{1}{x^2+x}\, dx$.

5. (a) Write down the Taylor series for e^x.

   (b) Express the indefinite integral

   $$\int \frac{e^x - 1}{x}\, dx$$

   as a power series.

   (c) Find the definite integral as a series,

   $$\int_0^2 \frac{e^x - 1}{x}\, dx$$

6. (a) Write down a power series centered at 0 or the MacLaurin series for
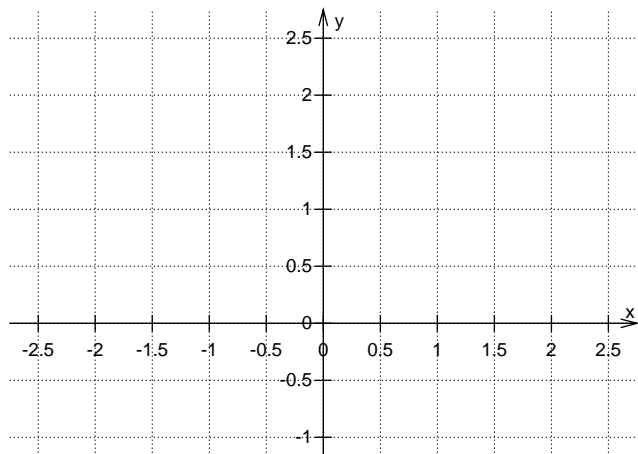
   $$\frac{1}{1+2x}.$$

   (b) Find the MacLaurin series for

   $$\frac{1}{(1+2x)^2}.$$

   Give the interval of convergence for this series.

7. (a) Sketch the curve $x(t) = t^2$, $y(t) = t^3 - t$.

(b) Find the coordinates of the point where the curve crosses itself.

(c) Find the equations of the tangent lines at this point.

8. (a) Sketch the curve given in polar coordinates by $r = 1 + 2\cos\theta$.

   (b) Find a range of θ which gives the inner loop in the graph from part a).