MA 114 — Calculus II Spring 2012
Final Exam April 30, 2012

- Each question is followed by space to write your answer. Write your solutions neatly in the space below the question.
- Clearly indicate your answer and the reasoning used to arrive at that answer. *Unsupported answers may not receive credit.*
- Unless a problem specifically asks for an approximation, you must give exact answers to recieve credit.
- You may use a calculator, but not one which has symbolic manipulation capabilities.
- Turn off your cell phones, and any other electronic devices which can send and receive wireless signals. You may not wear ear-plugs during the exam.
- No books or notes may be used.

Name: ______________________

Section: __________

Last four digits of student identification number: __________

$\sin^2 A + \cos^2 A = 1$
$1 + \cot^2 A = \csc^2 A$
$\tan^2 A + 1 = \sec^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(2A) = 2 \sin A \cos A$
$\cos(2A) = \cos^2 A - \sin^2 A$

Question	Score	Total
1		14=7+7
2		14=7+7
3		10=4+6
4		6=6
5		6=6
6		12=4+8
7		12=6+6
8		12=6+6
9		14=7+7
		100

1. (a) Evaluate the series $\displaystyle\sum_{n=2}^{\infty} \frac{4}{n^2-1}$

(b) Determine whether or not the series $\displaystyle\sum_{k=0}^{\infty} \frac{\cos k}{e^k}$ converges. Make sure to state the test(s) that you use, and verify that their assumptions are satisfied.

2. Let R be the (unbounded) region which lies below the curve $y = x^{-1.5}$, above the x-axis, and to the right of the line $x = 1$. Hint: For the problems below, think of R as being bounded on the right by the line $x = a$ for some large a, and then let $a \to \infty$.

   (a) Consider the solid obtained by revolving R about the x-axis. Determine whether or not this solid has finite volume. If it does, compute it. Make sure to clearly indicate the relevant integral.

   (b) Consider the solid obtained by revolving R about the y-axis. Determine whether or not this solid has finite volume. If it does, compute it. Make sure to clearly indicate the relevant integral.

3. An oddly-shaped well is 50 feet deep, and water (which weighs 62 pounds per cubic foot) fills the bottom 40 feet. Let $A = A(x)$ be the cross-sectional area (in square feet) of the well with respect to the height x (in feet) from the bottom of the well.

   (a) The work required to empty the well is given by an integral of the form

   $$\int_a^b c\,(r - x)\,A(x)\,dx\ .$$

   Give the values of the constants a, b, c and r.

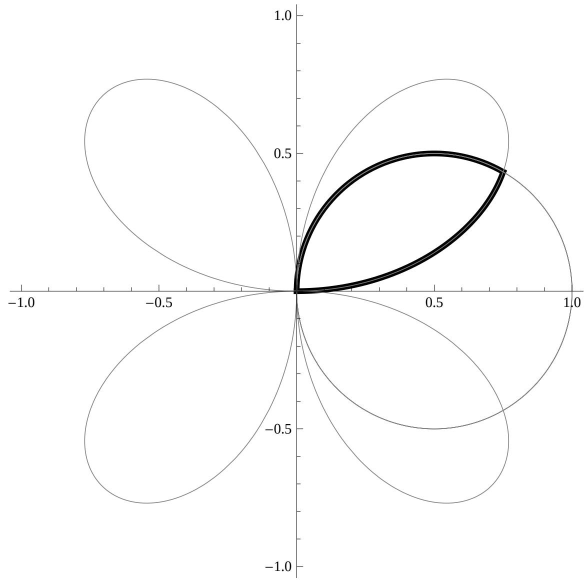
   (b) Use Simpson's rule and the measurments below to estimate the work needed to pump all of the water from the well.

x	0	10	20	30	40	50
A	30	25	30	25	30	25

4. Give the Taylor polynomial of degree 2, centered at 0, for the function $f(x) = (2 - x)^{\pi}$.

5. Set up an integral for evaluating the arclength of the cycloid $x = t - \sin t$, $y = 1 - \cos t$ for $0 \leq t \leq 2\pi$. Simplify the integrand using basic trigonometric identities. *Do not evaluate this integral.*

6. Consider the polar curves $r = \sin(2\theta)$ and $r = \cos\theta$, which are pictured below.



(a) Determine the Cartesian coordinates (x, y) of the point of intersection which is strictly in the first quadrant, i.e. $x, y > 0$.

(b) Set up an integral, or integrals, for computing the area of the region in the first quadrant between the bolded portion of the two curves. *Do not evaluate the integral(s).*

7. The Bessel function J_0 is given by the power series $J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left(\frac{x}{2}\right)^{2n}$.

(a) Determine the interval of convergence for this series.

(b) Letting S_N denote the N^{th} partial sum, determine the smallest integer N for which it is guaranteed that $|J_0(1/2) - S_N(1/2)| < 10^{-16}$.

8. Consider the seasonal-growth model $\frac{dP}{dt} = kP\cos(rt)$, $P(0) = P_0$, where k, r and P_0 are positive constants.

(a) Give a formula for P in terms of t, k, r and P_0.

(b) Taking the values $r = 2$, $k = 1$ and $P_0 = 100$, estimate $P(\pi/2)$ using Euler's method, with step-size $h = \pi/4$. You may give your answer as a decimal approximation (providing at least three digits beyond the decimal).

9. Evaluate each of the definite or indefinite integrals below:

   (a) $\int x \arctan x \, dx$

   (b) $\int_0^{\pi/4} \tan^3 x \, \sec^3 x \, dx$