STA 291
Lecture 11

- **Describing Quantitative Data**
  – Measures of Central Location

  Examples of mean and median

  – Review of Chapter 5. using the probability rules

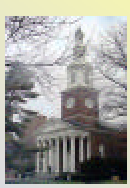
- You need a Calculator for the exam, but ***no*** laptop, ***no*** cellphone, ***no*** blackberry, ***no*** iphone, etc (anything that can transmitting wireless signal is not allowed)

- Location: Memorial Hall,
- Time: Tuesday 5-7pm.

- Talk to me if you have a conflict.

- A Formula sheet, with probability rules and sample mean etc will be available.

- Memorial Hall



- Feb. 23 5-7pm
- Covers up to mean and median of a sample (beginning of chapter 6). But not any measure of spread (i.e. standard deviation, inter-quartile range etc)

Chapter 1-5, 6(first 3 sections) + 23(first 5 sections)

Summarizing Data Numerically

- Center of the data
  - Mean (average)
  - Median
  - Mode (…will not cover)
- Spread of the data
  - Variance, Standard deviation
  - Inter-quartile range
  - Range

Mathematical Notation: Sample Mean

- Sample size n
- Observations $x_1, x_2, \ldots, x_n$
- Sample Mean "x-bar" --- a statistic

$$\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i$$

$\sum =$ SUM

Mathematical Notation: Population Mean for a finite population of size *N*

- Population size (finite) N
- Observations $x_1, x_2, \ldots, x_N$
- Population Mean "mu" --- a Parameter

$$m = (x_1 + x_2 + \ldots + x_N) / N$$
$$= \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\sum = \text{SUM}$$

Infinite populations

- Imagine the population mean for an infinite population.
- Also denoted by mu or m
- Cannot compute it (since infinite population size) but such a number exist in the limit.
- Carry the same information.

Infinite population

- When the population consists of values that can be ordered
- Median for a population also make sense: it is the number in the middle….half of the population values will be below, half will be above.

Mean

- If the distribution is highly skewed, then the mean is not representative of a typical observation
- Example:

  Monthly income for five persons

  1,000 2,000 3,000 4,000 100,000
- Average monthly income: = 22,000
- Not representative of a typical observation.

- Median = 3000

Median

- The median is the measurement that falls in the middle of the *ordered* sample
- When the sample size *n* is odd, there is a middle value
- It has the ordered index *(n+1)/2*
- Example: 1.1, 2.3, 4.6, 7.9, 8.1

  n=5, (n+1)/2=6/2=3, so index = 3,

  Median = 3rd smallest observation = 4.6

Median

- When the sample size *n* is even, average the two middle values
- Example: 3, 7, 8, 9, $n=4$,

$(n+1)/2=5/2=2.5$, *index* = 2.5

Median = midpoint between 2nd and 3rd smallest observation = (7+8)/2 =7.5

STA 291 - Lecture 11 13

Summary: Measures of Location

Mean- Arithmetic Average

Mean of a Sample - $\bar{x}$

Mean of a Population - μ

Median – Midpoint of the observations when they are arranged in increasing order

Notation: Subscripted variables
n = # of units in the sample
N = # of units in the population
x = Variable to be measured
x_i = Measurement of the *ith* unit

Mode….

STA 291 - Lecture 11 14

Mean vs. Median

Observations	Median	Mean
1, 2, 3, 4, 5	3	3
1, 2, 3, 4, 100	3	22
3, 3, 3, 3, 3	3	3
1, 2, 3, 100, 100	3	41.2

STA 291 - Lecture 11 15

Mean vs. Median

- If the distribution is symmetric, then Mean=Median
- If the distribution is skewed, then the mean lies more toward the direction of skew
- Mean and Median Online Applet

STA 291 - Lecture 11 16

Example

- the sample consist of 5 numbers, 3.6, 4.4, 5.9, 2.1, and the last number is over 10.

(some time we write it as 10+)

- Median = 4.4

- Can we find the mean here? No

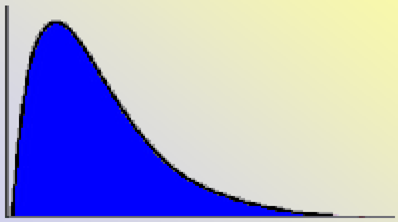
STA 291 - Lecture 11 17

Example: Mean and Median

- Example: Weights of forty-year old men
  158, 154, 148, 160, 161, 182,
  166, 170, 236, 195, 162
- Mean =
- Ordered weights: (order a large dataset can take a long time)
- 148, 154, 158, 160, 161, 162,
  166, 170, 182, 195, 236
- Median = 162

STA 291 - Lecture 11 18

Eye ball the plot to find mean/median

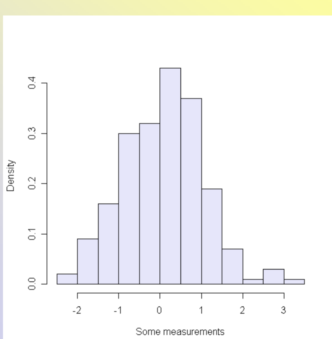


STA 291 - Lecture 11 19

- Extreme valued observations pulls mean, but not on median.

For data with a symmetric histogram, mean=median.

STA 291 - Lecture 11 20



STA 291 - Lecture 11 21

Using probability rule

- In a typical week day, a restaurant sells ? Gallons of house soup.

- Given that

P(sell more than 5 gallon) = 0.8

P(sell less than 10 gallon) = 0.7

- P(sell between 5 and 10 gallon) = 0.5

Why not always Median?

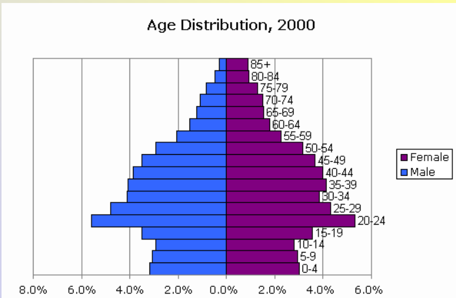
- Disadvantage: Insensitive to changes **within** the lower or upper half of the data
- Example: 1, 2, 3, 4, 5, 6, 7 vs.

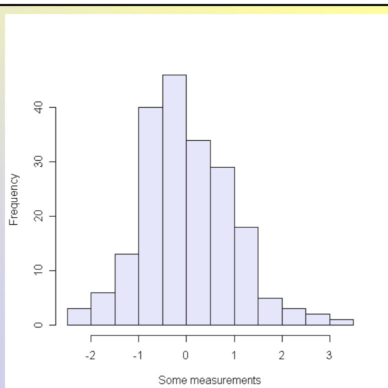
  1, 2, 3, 4, 100,100,100
- For symmetric, bell shaped distributions, mean is more informative.
- Mean is easy to work with. Ordering can take a long time
- *Sometimes*, the mean is more informative even when the distribution is slightly skewed

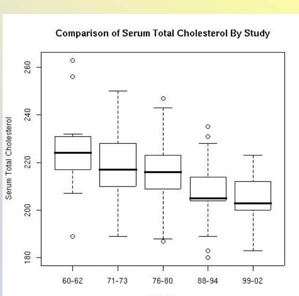
Census Data	Lexington	Fayette County	Kentucky	United States
Population	261,545	261,545	4,069,734	281,422,131
Area in square miles	306	306	40,131	3,554,141
People per sq. mi.	853	853	101	79
Median Age	35	34	36	36
Median Family Income	$42,500	$39,500	$32,101	$40,591

Real Estate Market Data	Lexington	Fayette County	Kentucky	United States
Total Housing Units	54,587	54,587	806,524	115,904,743
Average Home Price	$151,776	$151,776	$115,545	$173,585
Median Rental Price	$383	$383	$257	$471
Owner Occupied	52%	52%	64%	60%

Given a histogram, find approx mean and median







Five-Number Summary

- Maximum, Upper Quartile, Median, Lower Quartile, Minimum
- Statistical Software SAS output
  (Murder Rate Data)

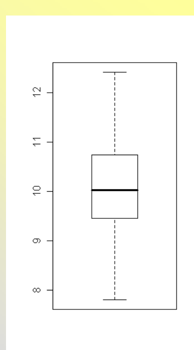
Quantile	Estimate
100% Max	20.30
75% Q3	10.30
50% Median	6.70
25% Q1	3.90
0% Min	1.60

Five-Number Summary

- Maximum, Upper Quartile, Median, Lower Quartile, Minimum
- Example: The five-number summary for a data set is min=4, Q1=256, median=530, Q3=1105, max=320,000.
- What does this suggest about the shape of the distribution?

Box plot

- A box plot is a graphic representation of the five number summary --- provided the max is within 1.5 IQR of Q3 (min is within 1.5 IQR of Q1)



Attendance Survey Question

- On a 4”x6” index card
  - write down your name and section number
  - Question:

Pick one: Mean or Median

_______ is a measure more resistant to extreme valued observations in the sample.