## STA 291
Lecture 13, Chap. 6

- **Describing Quantitative Data**
  - Measures of Central Location
  - Measures of Variability (spread)



## Summarizing Data Numerically

- Center of the data
  - Mean (average)
  - Median
  - Mode (…will not cover)
- Spread of the data
  - Variance, Standard deviation
  - Inter-quartile range
  - Range



## Mathematical Notation: Sample Mean

- Sample size $n$
- Observations $x_1, x_2, \ldots, x_n$
- Sample Mean "x-bar" --- a statistic

$$\bar{x} = (x_1 + x_2 + \ldots + x_n)/n$$

$$= \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sum = \text{SUM}$$

## Mathematical Notation: Population Mean for a finite population of size *N*

- Population size (finite) $N$
- Observations $x_1, x_2, \ldots, x_N$
- Population Mean "mu" – a Parameter

$$\mu = (x_1 + x_2 + \ldots + x_N) / N$$

$$= \frac{1}{N} \sum_{i=1}^{N} x_i$$

$$\sum = \text{SUM}$$



## Percentiles

- The *p*th percentile is a number such that *p*% of the observations take values below it, and (100-*p*)% take values above it
- 50th percentile = median
- 25th percentile = lower quartile
- 75th percentile = upper quartile



## Quartiles

- 25th percentile = lower quartile = Q1
- 75th percentile = upper quartile = Q3

**Interquartile range** = Q3 - Q1
(a measurement of variability in the data)

## SAT Math scores

- Nationally (min = 210 max = 800 )
  Q1 = 440
  Median = Q2 = 520
  Q3 = 610 ( -- you are better than 75% of all test takers)

- Mean = 518 (SD = 115 what is that?)



## SAT Percentile Ranks

**Critical Reading, Mathematics, and Writing**

| Score | Critical Reading | Mathematics | Writing |
|---|---|---|---|
| 800 | 99 | 99 | 99+ |
| 790 | 99 | 99 | 99+ |
| 780 | 99 | 99 | 99 |
| 770 | 99 | 99 | 99 |
| 760 | 99 | 98 | 99 |
| 750 | 98 | 98 | 99 |
| 740 | 98 | 97 | 98 |
| 730 | 97 | 97 | 98 |
| 720 | 96 | 96 | 97 |
| 710 | 96 | 95 | 97 |
| 700 | 95 | 93 | 96 |
| 690 | 94 | 92 | 95 |
| 680 | 93 | 91 | 94 |
| 670 | 92 | 89 | 93 |
| 660 | 90 | 88 | 92 |
| 650 | 89 | 86 | 90 |
| 640 | 87 | 83 | 89 |
| 630 | 85 | 81 | 87 |
| 620 | 83 | 79 | 85 |
| 610 | 82 | 76 | 83 |
| 600 | 79 | 74 | 81 |
| 590 | 77 | 71 | 79 |
| 580 | 74 | 68 | 76 |
| 570 | 71 | 66 | 73 |
| 560 | 68 | 63 | 71 |
| 550 | 65 | 60 | 68 |
| 540 | 62 | 56 | 64 |
| 530 | 58 | 53 | 62 |
| 520 | 55 | 50 | 58 |
| 510 | 51 | 47 | 54 |
| 500 | 48 | 43 | 51 |
| 490 | 44 | 40 | 47 |
| 480 | 41 | 36 | 44 |
| 470 | 37 | 33 | 40 |
| 460 | 34 | 30 | 37 |

## Five-Number Summary

- Maximum, Upper Quartile, Median, Lower Quartile, Minimum
- Statistical Software SAS output
  (Murder Rate Data)

```
Quantile       Estimate

100% Max         20.30
75% Q3           10.30
50% Median        6.70
25% Q1            3.90
0% Min            1.60
```

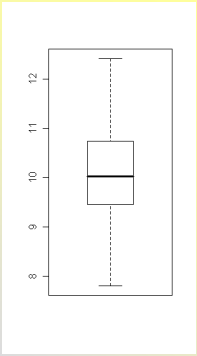
## Five-Number Summary

- Maximum, Upper Quartile, Median, Lower Quartile, Minimum
- Example: The five-number summary for a data set is min=4, Q1=256, median=530, Q3=1105, max=320,000.
- What does this suggest about the shape of the distribution?



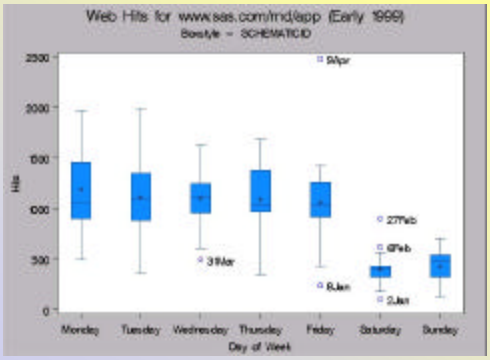
## Box plot

- A box plot is a graphic representation of the five number summary --- provided the max is within 1.5 IQR of Q3 (min is within 1.5 IQR of Q1)






- Otherwise the max (min) is suspected as an **outlier** and treated differently.






- Box plot is most useful when compare several populations



## Measures of Variation

- Mean and Median only describe the central location, but not the spread of the data
- Two distributions may have the same mean, but different variability
- Statistics that describe variability are called measures of spread/variation

## Measures of Variation

- Range: = max - min
  Difference between maximum and minimum value
- Variance: $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$
- 
- Standard Deviation: $s = \sqrt{s^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$
- Inter-quartile Range: = Q3 – Q1
  Difference between upper and lower quartile of the data



## Deviations: Example

- Sample Data: 1, 7, 4, 3, 10
- Mean (x-bar): (1+7+4+3+10)/5 =25/5=5

| data | Deviation | Dev. square |
|---|---|---|
| 1 | (1 - 5)= -4 | 16 |
| 3 | (3 - 5)= -2 | 4 |
| 4 | (4 - 5) = -1 | 1 |
| 7 | (7 - 5) = 2 | 4 |
| 10 | (10 - 5) = 5 | 25 |
| Sum=25 | Sum = 0 | sum = 50 |



## Sample Variance

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

The variance of *n* observations is the sum of the squared deviations, divided by *n-1.*

## Variance: Example

| Observation | Mean | Deviation | Squared Deviation |
|---|---|---|---|
| 1 | 5 | | 16 |
| 3 | 5 | | 4 |
| 4 | 5 | | 1 |
| 7 | 5 | | 4 |
| 10 | 5 | | 25 |
| Sum of the Squared Deviations | | | 50 |
| *n-1* | | | 5-1=4 |
| Sum of the Squared Deviations / *(n-1)* | | | 50/4=12.5 |



- So, sample variance of the data is 12.5
- Sample standard deviation is 3.53

$$\sqrt{12.5} = 3.53$$



- Variance/standard deviation is also more susceptible to extreme valued observations.
- We are using x-bar and variance/standard deviation mostly in the rest of this course.

## Population variance/standard deviation

- Notation for Population variance/standard deviation (usually obtain only after a census)
- Sigma-square / sigma

$s^2$ $s$



## standardization

- Describe a value in a sample by
- "how much standard deviation above/below the average"
- The value 6 is one standard deviation above mean -- the value 6 corresponds to a z-score of 1
- May be negative (for below average)



## Attendance Survey Question

- On a 4"x6" index card
  - write down your name and section number
  - Question: Independent or not?
  - Gender of first child and second child from same couple.