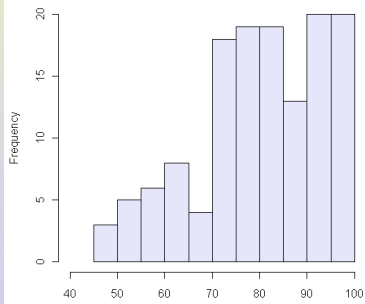


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Lecture 15

– Normal Distributions (Bell curve)

Exam 1 score



- Mean = 80.98
- Median = 82
- SD = 13.6

- Five number summary:

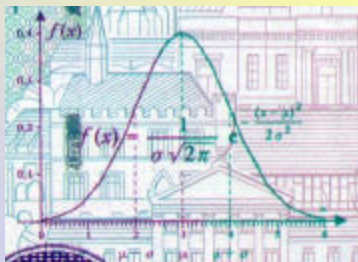
- 46 74 82 92 100

- There are many different shapes of continuous probability distributions...
- We focus on one type – the **Normal** distribution, also known as Gaussian distribution or bell curve.

Carl F. Gauss



Bell curve



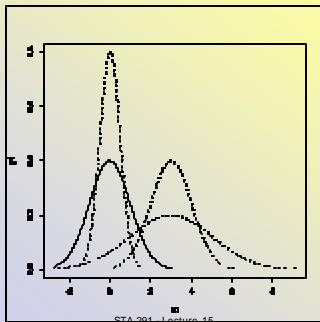
Normal distributions/densities

- Again, this is a whole family of distributions, indexed by mean and SD. (location and scale)

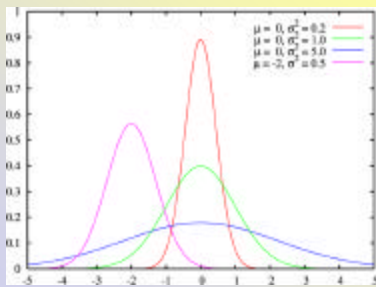
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Different Normal Distributions



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The Normal Probability Distribution

- Normal distribution is perfectly **symmetric** and **bell-shaped**
- Characterized by **two parameters**: **mean μ** and **standard deviation s**
- The **68%-95%-99.7%** rule applies to the normal distribution.
- That is, the probability concentrated within 1 standard deviation of the mean is always 68%; within 2 SD is 95%; within 3 SD is 99.7% etc.

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- It is very common.
- The sampling distribution of many common statistics are approximately Normally shaped, when the sample size n gets large.

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Central Limit Theorem

- In particular:
- Sample proportion \hat{p}
- Sample mean \bar{X}
- The sampling distribution of both will be approximately **Normal**, for **large n**

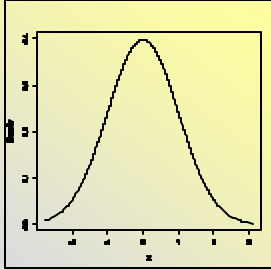
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Standard Normal Distribution

- The standard normal distribution is the normal distribution with mean $\mu=0$ and standard deviation

$$s = 1$$



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Non-standard normal distribution

- Either mean $m \neq 0$
- Or the SD $s \neq 1$
- Or both.
- In real life the normal distribution are often non-standard.

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Examples of normal random variables

- Public demand of gas/water/electricity in a city.
- Amount of Rain fall in a season.
- Weight/height of a randomly selected adult female.

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Examples of normal random variables – cont.

- Soup sold in a restaurant in a day.
- Stock index value tomorrow.

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Example of *non-normal* probability distributions

Income of a randomly selected family.
(skewed, only positive)

Price of a randomly selected house.
(skewed, only positive)

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Example of non-normal probability distributions

- Number of accidents in a week. (discrete)
- Waiting time for a traffic light. (has a discrete value at 0, and only with positive values, and no more than 3min, etc)

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Central Limit Theorem

- Even the incomes are not normally distributed, the **average** income of many randomly selected families **is** approximately normally distributed.
- Average does the magic of making things normal! (transform to normal)

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Table 3 is for **standard normal**

- Convert non-standard to standard.
- Denote by X -- non-standard normal
- Denote by Z -- standard normal

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Standard Normal Distribution

- When values from an arbitrary normal distribution are converted to z-scores, then they have a standard normal distribution
- The conversion is done by subtracting the mean μ , and then dividing by the standard deviation σ

$$z = \frac{x - \mu}{\sigma}$$

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Example

- Find the probability that a randomly selected female adult height is between the interval 161cm and 170cm. Recall

$$m = 165, s = 8$$

$$\frac{161 - 165}{8} = -0.5;$$

$$\frac{170 - 165}{8} = 0.625$$

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Example –cont.

- Therefore the probability is the same as a standard normal random variable Z between the interval -0.5 and 0.625

$$P(161 < X < 170) = P(-0.5 < Z < 0.625)$$

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Use table or use Applet?

- Applet 1:
http://bcs.whfreeman.com/scc/content/cat_040/spt/normalcurve/normalcurve.html
- Applet 2:
<http://psych-www.colorado.edu/~mcclella/java/normal/handleNormal.html>

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Online Tool

- [Normal Density Curve](#)
- Use it to verify graphically the empirical rule, find probabilities, find percentiles and z-values for one- and two-tailed probabilities

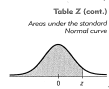


Table Z (cont.)

Tables and Selected Formulas A-77

z	Second decimal place in z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9939	0.9940	0.9941	0.9942	0.9943	0.9944	0.9945	0.9945	0.9946
2.6	0.9946	0.9947	0.9947	0.9948	0.9948	0.9949	0.9949	0.9949	0.9949	0.9949
2.7	0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9949	0.9949

z-Scores

- The z-score for a value x of a random variable is the number of standard deviations that x is above μ
- If x is below μ , then the z-score is negative
- The z-score is used to compare values from different normal distributions

Calculating z-Scores

- You need to know x , μ , and S to calculate z

$$z = \frac{x - \mu}{S}$$

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- Applet does the conversion automatically. (recommended)

The table 3 gives probability

$$P(0 < Z < z) = ?$$

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Tail Probabilities

- SAT Scores: Mean=500, SD=100
- The SAT score 700 has a z-score of $z=2$
- The probability that a score is **beyond** 700 is the tail probability of Z **beyond** 2

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z-Scores

- The z-score can be used to compare values from different normal distributions
- SAT: $\mu=500$, $s=100$
- ACT: $\mu=18$, $s=6$
- Which is better, 650 in the SAT or 26 in the ACT?

$$z_{SAT} = \frac{x - \mathbf{m}}{\mathbf{s}} = \frac{650 - 500}{100} = 1.5$$

$$z_{ACT} = \frac{x - \mathbf{m}}{\mathbf{s}} = \frac{26 - 18}{6} = 1.333$$

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- Corresponding tail probabilities?
- How many percent of total test scores have better SAT or ACT scores?

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Typical Questions

1. Probability (right-hand, left-hand, two-sided, middle)
2. z-score
3. Observation (raw score)

- To find probability, use applet or Table 3.
- In transforming between 2 and 3, you need mean and standard deviation

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Finding z-Values for Percentiles

- For a normal distribution, how many standard deviations from the mean is the 90th percentile?
- What is the value of z such that 0.90 probability is less than $\mu + z s$?
- If 0.9 probability is less than $\mu + z s$, then there is 0.4 probability between 0 and $\mu + z s$ (because there is 0.5 probability less than 0)
- $z=1.28$
- The 90th percentile of a normal distribution is 1.28 standard deviations above the mean

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Quartiles of Normal Distributions

- Median: $z=0$
(0 standard deviations above the mean)
- Upper Quartile: $z = 0.67$
(0.67 standard deviations above the mean)
- Lower Quartile: $z = -0.67$
(0.67 standard deviations below the mean)

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- In fact for any normal probability distributions, the 90th percentile is always 1.28 SD above the mean

the 95th percentile is ____ SD above mean

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Finding z-Values for Two-Tail Probabilities

- What is the z-value such that the probability is 0.1 that a normally distributed random variable falls more than z standard deviations **above or below** the mean
- Symmetry: we need to find the z-value such that the right-tail probability is 0.05 (more than z standard deviations **above** the mean)
- $z=1.65$
- 10% probability for a normally distributed random variable is outside 1.65 standard deviations from the mean, and 90% is within 1.65 standard deviations from the mean

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homework online

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Attendance Survey Question 16

- On a 4"x6" index card
 - Please write down your name and section number
 - Today's Question:

___?___ is also been called bell curve.

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