

STA 291

Lecture 22

- Chapter 11 Testing Hypothesis
 - Concepts of Hypothesis Testing

- **Bonus** Homework, due in the lab April 20-22:
Essay "How would you test the 'hot hand' theory in basketball games?" (~400-600 words / approximately one typed page)
- Be as specific as you can: what data to collect? how many cases to collect? What hypothesis you are testing?

Significance Tests

- A significance test checks whether data agrees with a (null) **hypothesis**
- A hypothesis is a statement about a characteristic of a population parameter or parameters
- If the data is very unreasonable under the hypothesis, then we will **reject** the hypothesis
- Usually, we try to find evidence **against** the hypothesis

Logical Procedure

1. State a (null) hypothesis that you would like to find evidence against
2. Get data and calculate a statistic (for example: sample proportion)
3. The hypothesis (and CLT) determines the sampling distribution of our statistic
4. If the calculated value in 2. is very unreasonable given 3 (i.e. almost impossible), then we conclude that the hypothesis was wrong

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4

Example 1

- Somebody makes the claim that "Nicotine Patch and Zyban has same effect on quitting smoke"
- You don't believe it. So you conduct the experiment and collect data: Patch: 244 subjects; 52 quit. Zyban: 244 subjects; 85 quit.
- How (un)likely is this under the *hypothesis* of no difference?
- The sampling distribution helps us quantify the (un)likeliness in terms of a probability (p -value)

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5

Example 2

- Mr. Basketball was an 82% free throw shooter last season. This season so far in 59 free throws he only hit 40.
- (null) Hypothesis: He is still an 82% shooter
- alternative hypothesis: his percentage has changed. (not 82% anymore)

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6

Question:

- How unlikely are we going to see 52/244 verses 85/244 if indeed Patch and Zyban are equally effective? (Probability = ?)
- How unlikely for an 82% shooter to hit only 40 out of 59? (Probability = ?)

How small is too small?

- A small probability imply very unlikely or impossible. (No clear cut, but Prob less than 0.01 is certainly small)
- A larger probability imply this is likely and no surprise. (again, no clear boundary, but prob. > 0.1 is certainly not small)

- For the Basketball data, we actually got Probability = 0.0045
- For the Patch vs. Zyban data, we actually got Probability = 0.0013

Usually we pick an alpha level

- Suppose we pick $\alpha = 0.05$, then Any probability below 0.05 is deemed “impossible” so this is evidence against the null hypothesis – we say that “we reject the null hypothesis”
- Otherwise, we say “we cannot reject the null hypothesis” imply there is not enough
- Evidence against the null hypothesis

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10

- Notice “not enough evidence against null hypothesis” is different from
- “validated the null hypothesis”, “accept null hypothesis”,
- It could mean there is simply not enough data to reach any conclusion.

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11

- If the basketball data were 14 hits out of 20 shoots ($14/20 = 0.7$), the P-value would be 0.16247.
- This probability is not small.
- Usually we cut off (that's the alpha level) at 0.05 or 0.01 for P-values

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12

Significance Test

- A **significance test** is a way of statistically testing a hypothesis by comparing the data to values predicted by the hypothesis
- Data that fall far from the predicted values provide **evidence against the hypothesis**

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13

Elements of a Significance Test

- **Assumptions** (about population dist.)
- **Hypotheses** (about popu. Parameter. null and alternative)
- **Test Statistic** (based on a SRS.)
- **P-value** (a way of summarizing the strength of evidence.)
- **Conclusion** (reject, or not reject, that is the question)

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14

Assumptions

- What type of data do we have?
 - Qualitative or quantitative?
 - Different types of data require different test procedures
 - If we are comparing 2 population means, then how the SD differ?
- What is the population distribution?
 - Is it normal? Or is it binomial?
 - Some tests require normal population distributions (t-test)

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15

Assumptions-cont.

- Which sampling method has been used?
 - We usually assume Simple Random Sampling
- What is the sample size?
 - Some methods require a minimum sample size (like $n > 30$) because of using CLT

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16

Assumptions in the Example1

- What type of data do we have?
 - Qualitative with two categories:
Either “quit smoke” or “not quit smoke”
- What is the population distribution?
 - It is Bernoulli type. It is definitely not normal since it can only take two values
- Which sampling method has been used?
 - We assume simple random sampling
- What is the sample size?
 - $n=244$

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17

Hypotheses

- Hypotheses are statements about population parameter.
- The **null hypothesis (H_0)** is the hypothesis that we test (and try to find evidence against)
- The name null hypothesis refers to the fact that it often (not always) is a hypothesis of “no effect” (no effect of a medical treatment, no difference in characteristics of populations, etc.)

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18

- The **alternative hypothesis (H_1)** is a hypothesis that contradicts the null hypothesis
- When we reject the null hypothesis, we are in favor of the alternative hypothesis.
- Often, the alternative hypothesis is the actual research hypothesis that we would like to “prove” by finding evidence against the null hypothesis (proof by contradiction)

Hypotheses in the Example 1

- **Null hypothesis (H_0):**
The percentage of quitting smoke with Patch and Zyban are the same
 $H_0: \text{Prop(patch)} = \text{Prop(zyban)}$
- **Alternative hypothesis (H_1):**
The two proportions differ

Hypotheses in the Example 2

- **Null hypothesis (H_0):**
The percentage of free throw for Mr. Basketball is still 82%
 $H_0: \text{Prop} = 0.82$
- **Alternative hypothesis (H_1):**
The proportion differs from 0.82

Test Statistic

- The **test statistic** is a statistic that is calculated from the sample data
- Formula will be given for test statistic, but you need to choose the right one.

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22

Test Statistic in the Example 2

- **Test statistic:**
Sample proportion, $\hat{p} = 40/59 = 0.6779$

$$z_{obs} = \frac{\hat{p} - 0.82}{\sqrt{0.82(1 - 0.82)/59}}$$

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23

p-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?
- The **p-value** is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed
- The smaller the p-value, the more strongly the data contradict H_0

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24

Conclusion

- Sometimes, in addition to reporting the p -value, a formal decision is made about rejecting or not rejecting the null hypothesis
- Most studies require small p -values like $p < .05$ or $p < .01$ as significant evidence against the null hypothesis
- “The results are significant at the 5% level”

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25

p -Values and Their Significance

- p -Value < 0.01 :
Highly Significant / “Overwhelming Evidence”
- $0.01 < p$ -Value < 0.05 :
Significant / “Strong Evidence”
- $0.05 < p$ -Value < 0.1 :
Not Significant / “Weak Evidence”
- p -Value > 0.1 :
Not Significant / “No Evidence”

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26

Decisions and Types of Errors in Tests of Hypotheses

- Terminology:
 - The alpha-level (significance level) is a number such that one rejects the null hypothesis if the p -value is less than or equal to it. The most common alpha-levels are .05 and .01
 - The choice of the alpha-level reflects how cautious the researcher wants to be
 - The significance level needs to be chosen **before** analyzing the data

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27

Decisions and Types of Errors in Tests of Hypotheses

- More Terminology:
 - The rejection region is a range of values such that if the test statistic falls into that range, we decide to reject the null hypothesis in favor of the alternative hypothesis

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28

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.
- Type II Error: The null hypothesis is not rejected, even though it is false.

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29

Type I and Type II Errors

		Decision	
		Reject	Do not reject
Condition of the null hypothesis	True	Type I error	Correct
	False	Correct	Type II error

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30

Type I and Type II Errors

- Terminology:
 - **Alpha** = Probability of a Type I error
 - **Beta** = Probability of a Type II error
 - **Power** = $1 - \text{Probability of a Type II error}$
- The smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you ask for very strong evidence to reject the null hypothesis, it is more likely that you fail to detect a real difference

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31

Type I and Type II Errors

- In practice, alpha is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult
- **How to choose alpha?**
- If the consequences of a Type I error are very serious, then alpha should be small.
- For example, you want to find evidence that someone is guilty of a crime
- In exploratory research, often a larger probability of Type I error is acceptable
- If the sample size increases, both error probabilities can decrease

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32

Attendance Survey Question 23

- On a 4"x6" index card
 - Please write down your name and section number
 - Today's Question:
 - What is "alpha-level" (in hypothesis testing)

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33
