STA 291
Lecture 24

- *Two kinds of Error in Testing hypothesis*
- *Examples.*

About **bonus** project

Due in Lab April 20- 22

- Some survey show a majority people believe in "hot hand". A follow-up question then is: if there were "hot hand", how much better/worse a shooter can become by the previous shoots? (i.e. what is a reasonable difference to expect)
- i.e. whether he made or missed the two previous shots, how much difference do you think this has the effect on the present shoot? (in terms of hitting percentages) 5%? 10%? or even 20%?

- Since a small difference will need more data to detect.
- A larger difference can be discovered with less data.

- Some clue: margin of error calculations.

- Bonus is worth equivalent to one LAB
- Lab will start to give “practical quiz”

Decisions and Types of Errors in Tests of Hypotheses

- Terminology:
  - The alpha-level (significance level) is a *threshold number* such that one rejects the null hypothesis if the *p*-value is less than or equal to it. The most common alpha-levels are 0.05 and 0.01
  - The choice of the alpha-level reflects how cautious the researcher wants to be (when it come to reject null hypothesis)

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.
- Type II Error: The null hypothesis is not rejected, even though it is false.

- Setting the alpha-level low protect us from type I Error. (the probability of making a type I error is less than alpha)

Type I and Type II Errors

		Decision	
		Reject null	Do not reject null
the null hypothesis	True	***Type I error***	***Correct***
	False	***Correct***	***Type II error***

Type I and Type II Errors

- Terminology:
  - ***Alpha*** = Probability of make a Type I error
  - ***Beta*** = Probability of make a Type II error
  - ***Power*** = 1 – Probability of a Type II error = 1 - Beta
- For a given data, the smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- i.e. If you set alpha very small, it is more likely that you fail to detect a real difference (larger Beta).

- When sample size(s) increases, both error probabilities could be made to decrease.

- Our Strategy:

  -- keep type I error probability small by pick a small alpha.

  -- Increase sample size to make Beta small.

- Depend on how expensive to obtain data, a Beta = 0.15 is not uncommon.

Type I and Type II Errors

- In practice, alpha is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult (sample size calculation)
- **How to choose alpha?**
- If the consequences of a Type I error are very serious, then chose a smaller alpha, like 0.01.
- For example, you want to find evidence that someone is guilty of a crime.
- In exploratory research, often a larger probability of Type I error is acceptable (like 0.05 or even 0.1)

Example: New drug development

- The null hypothesis usually state that the new drug is “no difference” to the placebo.

- A type I error in this context is: falsely conclude a drug is useful when it is actually “NO effect”
- A type II error in this context is: falsely dismiss a useful drug.

Alternative and p-value computation

$$H_0 : p = p_0$$

	One-Sided Tests		Two-Sided Test
alternative Hypothesis	$H_A : p < p_0$	$H_A : p > p_0$	$H_A : p \neq p_0$
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

$$z_{obs} = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

Example

- Two consumer products (shampoo, laundry detergent etc) comparison. Call them A vs. B

- n consumers are given both products in the identical packaging. After one week of use of both products, state a preference.

- If there were no difference, then we should see 50%-50%

- Suppose in n=236 consumers, 110 prefer product A. Let p = popu. proportion prefer A. Use alpha = 0.05

- Null: $H_0 : p = 0.5$
- Alternative: $H_A : p \neq 0.5$

- Compute z_{obs}

- Sample proportion, $\hat{p}$ = 110/236 =0.4661

$$z_{obs} = \frac{0.4661 - 0.5}{\sqrt{0.5(1-0.5)/236}} = -1.04156$$

- Finally look the Z table for P-value:
- P-value=2P(Z>1.04)=2(1- 0.8508) =0.2984

- Conclusion, we do not reject null hypothesis since P-value is not less than alpha.
- Since 0.2984 is not less than 0.05

Two sample cases are similar, with two differences:

- Hypothesis involve 2 parameters from 2 populations
- Test statistic, z_{obs} , is different, involve 2 samples

Alternative and p-value computation

$$H_0 : p_1 - p_2 = 0$$

	One-Sided Tests		Two-Sided Test
alternative Hypothesis	$H_A : p_1 - p_2 < 0$	$H_A : p_1 - p_2 > 0$	$H_A : p_1 - p_2 \neq 0$
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

Two p's

$H_0 : p_1 = p_2$ which is equivalent to $H_0 : p_1 - p_2 = 0$,

$$z_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n_1} + \dfrac{\hat{p}(1-\hat{p})}{n_2}}}$$

- Where the $\hat{p}$ in the denominator is the combined (pooled) sample proportion.

= Total number of successes over total number of observations

So there are 3 different sample proportions: from sample one, from sample two and from both samples.

- P for P-value in a test hypothesis setting
- p for population proportion
-
- $\hat{p}$ for sample proportion
- p_0 for the hypothesized population proportion value

Attendance Survey Question 24

- On a 4"x6" index card
  - Please write down your name and section number
  - Today's Question:

  - Probability of making a type II error is denoted by:

a. Alpha b. Beta c. Power

Example: compare 2 proportions

- A nation wide study: an aspirin every other day can sharply reduce a man's risk of heart attack. (New York Times, reporting Jan. 27, 1987)

- Aspirin group: 104 Heart Att. in 11037
- Placebo group: 189 Heart Att. in 11034
- Randomized, double-blinded study

Example – cont.

- Let aspirin = group 1; placebo = group 2

  p1 = popu. proportion of Heart att. for group 1

  p2 = popu. proportion of Heart att. for group 2

$$H_0: p_1 = p_2 \quad \text{which is equivalent to} \quad H_0: p_1 - p_2 = 0$$

$$H_A: p_1 \neq p_2 \quad \text{or} \quad H_A: p_1 - p_2 \neq 0$$

Example – cont.

- We may use software to compute a p-value
- p-value = 7.71e-07 = 0.000000771

Or we can calculate by hand:

$$z_{obs} = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}}$$

Example – cont.

- n1= 11037, n2 = 11034

$$\hat{p}_1 = 104/11037 = 0.00942285$$

$$\hat{p}_2 = 189/11034 = 0.01712887$$

$$\hat{p} = (104 + 189)/(11037 + 11034) = 0.013275$$

z = - 0.00770602/0.001540777

= - 5.001386

$$z_{obs} = \frac{0.00942285 - 0.01712887}{\sqrt{\frac{\hat{p}(1-\hat{p})}{11037} + \frac{0.013275(1-0.013275)}{11034}}}$$

Example – cont.

- P-value= 2 x P(Z > | - 5.00|)
- It falls out of the range of our Z- table, so……

P-value is approx. zero. (much smaller than 0.0000?)

What is alpha level? Say it was 0.01. Since P-value is smaller than alpha, we reject the null hypothesis.