STA 291
Lecture 25

Testing the hypothesis about Population Mean

- ***Inference about a Population Mean, or compare two population means***

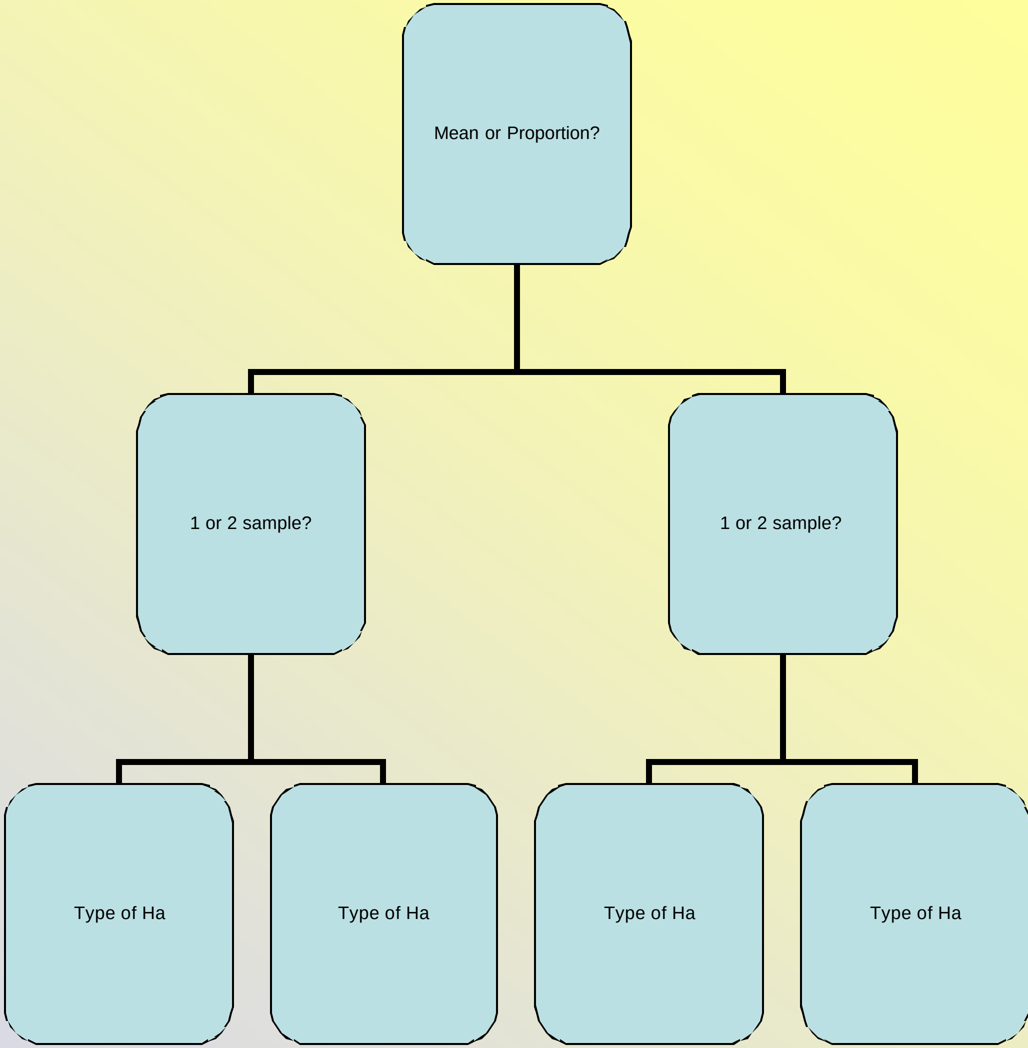
Which test?

- Tests about a population proportion (or 2 proportions) for population(s) with YES/NO type outcome.

- Tests about a population mean (or 2 means) for population(s) with continuous outcome. (normal or otherwise).
- If non-normal, we need large sample size

Different tests

- mean or proportion?
- 1 or 2 samples?
- Type of Ha ?



One or two samples

Compare the (equality of)

two proportions/two means ?

Or compare one proportion against a fixed number? (one mean against a fixed number?)

One-Sided Versus Two-Sided alternative hypothesis

- Two-sided hypothesis are more common
- Look for formulations like
  - “test whether the mean has ***changed***”
  - “test whether the mean has ***increased***”
  - “test whether the 2 means are ***the same***”
  - “test whether the mean has ***decreased***”
- Recall: Alternative hypothesis = research hypothesis

3 Alternatives about one population mean

$$H_0 : m = m_0$$

	One-Sided Tests		Two-Sided Test
Research Hypothesis	$H_A : m < m_0$	$H_A : m > m_0$	$H_A : m \neq m_0$
Test Statistic	$z_{obs} = \dfrac{\bar{X} - m_0}{s / \sqrt{n}}$		
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

Example

- The mean age at first marriage for married men in a New England community was 28 years in 1790.
- For a random sample of 406 married men in that community in 1990, the sample mean and standard deviation of age at first marriage were 26 and 9, respectively
- Q: Has the mean changed significantly?

Example –cont.

Hypotheses

- The null hypothesis $H_0 : m = 28$

  i.e. here $m_0 = 28$

- The alternative hypothesis is (recall the word changed?) $H_1 : m \neq 28$

Example – cont.

Test Statistic

$$z = \frac{\bar{X} - m_0}{s \ / \sqrt{n}}$$

- But this uses population SD , we do not have that and only have sample SD, s=9.
- When you use sample SD, you need to do student t-adjustment.

Example –cont.

- The test statistic should be

$$t = \frac{\bar{X} - m_0}{s / \sqrt{n}}$$

$$\frac{26-28}{9/\sqrt{406}} = -4.4776$$

$$2P(t > |-4.4776|) = 0.00000982$$

- This is by using software…….
- (2-sided) P-value
- Without a t-table software, we may use Z table as the df here is 405 (pretty big), the two tables are almost the same.

- If I were using the Z table,
- $2P(Z > 4.4776) = 2x(1 - 1.0000) = 0.0000$?
- (at least to 4 digits correct)

- A p-value of 0.0000? Or p-value of 0.00000982 will lead to the same conclusion:
- since it is way smaller than alpha=0.01, we reject the null hypothesis of “mean age = 28”

Example – cont.

In the exam we will say: use a significant level of 5% to make a decision. Etc

That is alpha

Example – cont.

- If t-table is not available (like in an exam), and sample size/df is over 100, use normal table (Z-table) to improvise. (with some small error)
- The p-value obtained is slightly smaller.

- Test the swimming/skiing/running etc timing after some equipment improvement.
- Usually the athlete is asked to try the new and old gears both and we shall record the differences. 0.5, 1.2, -0.06, ……0.66.
- Not a YES/NO outcome but a continuous one

- Seems to be a two sample? But if we look at the differences, there is only one difference

- Test about mean, one sample, two sided alternative hypothesis (population SD known)

$$H_0 : m = 0 \qquad \text{vs.} \qquad H_A : m \neq 0$$

a) Suppose z = - 2.6. Find the p-value. Does this provide strong, or weak, evidence against the null hypothesis?

Use table or applet to find p-value.

If sample SD were used we shall denote t = -2.7 etc

p-Value

- The ***p-value*** is the probability, assuming that H_0 is true, that the test statistic , z, takes values *at least as contradictory to* H_0 *as the value actually observed*

- *The **p-value** is **not** the probability that the hypothesis is true*

Small Sample Hypothesis Test for a Mean

- Assumptions
  - Quantitative variable, random sampling, **population distribution is normal**, ***any* sample size**
- Hypotheses
  - Same as in the large sample test for the mean

$$H_0 : m = m_0 \text{ vs. } H_1 : m \neq m_0$$

$$\text{or} \quad H_0 : m = m_0 \text{ vs. } H_1 : m > m_0$$

$$\text{or} \quad H_0 : m = m_0 \text{ vs. } H_1 : m < m_0$$

Hypothesis Test for a Mean

- Test statistic
  - Exactly the same as for the large sample test

$$t_{obs} = \frac{\bar{X} - m_0}{s / \sqrt{n}}$$

- *p*-Value
  - Same as for the large sample test (one-or two-sided), but using the web/applet for the *t* distribution
  - Table only provides very few values, almost un-useable.
- Conclusion
  - Report p-value and make formal decision

- Not going to exam on "computing p-value by using t-table when sample size/df is small.

- Either df > 100 so use the Z table instead.
- Or sigma is known so still use Z-table.
- Or the p-value will be given.
- So, the computation of p-value for us ….is always from Z table. (in reality could from t-table or other)

Hypothesis Test for a Mean: Example

- A study was conducted of the effects of a special class designed to improve children's verbal skills
- Each child took a verbal skills test twice, before and after a three-week period in the class
- X=2nd exam score – 1st exam score
- If the population mean for X, E(X)=mu equals 0, then the class has no effect
- Test the null hypothesis of no effect against the alternative hypothesis that the effect is positive
- Sample (*n*=*8*): 3,7,3,3.5, 0, -1, 2, 1

Normality Assumption

- An assumption for the *t*-test is that the population distribution is normal
- In practice, it is impossible to be 100% sure if the population distribution is normal
- It is useful to look at histogram or stem-and-leaf plot (or normal probability plot) to check whether the normality assumption is reasonable

Normality Assumption

- Good news: The *t*-test is relatively ***robust*** against violations of the assumption that the population distribution is normal
- Unless the population distribution is highly skewed, the *p*-values und confidence intervals are fairly accurate

- However: The random sampling assumption must never be violated, otherwise the test results are completely invalid

Decisions and Types of Errors in Tests of Hypotheses

- Terminology:
  - The alpha-level (significance level) is a number such that one rejects the null hypothesis if the *p*-value is less than or equal to it. The most common alpha-levels are .05 and .01
  - The choice of the alpha-level reflects how cautious the researcher wants to be

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.
- Type II Error: The null hypothesis is not rejected, even though it is false.

Type I and Type II Errors

		Decision	
		Reject null	Do not reject null
Condition of the null hypothesis	True	***Type I error***	***Correct***
	False	***Correct***	***Type II error***

Type I and Type II Errors

- Terminology:
  - ***Alpha*** = Probability of a Type I error
  - ***Beta*** = Probability of a Type II error
  - ***Power*** = 1 – Probability of a Type II error
- For a given data, the smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you need a very strong evidence to reject the null hypothesis (set alpha small), it is more likely that you fail to detect a real difference (larger Beta).

Type I and Type II Errors

- In practice, alpha is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult (sample size calculation)
- **How to choose alpha?**
- If the consequences of a Type I error are very serious, then chose a smaller alpha, like 0.01.
- For example, you want to find evidence that someone is guilty of a crime.
- In exploratory research, often a larger probability of Type I error is acceptable (like 0.05 or even 0.1)

Multiple Choice Question II

The P-value for testing the null hypothesis mu=100 (two-sided) is P-value=.001. This indicates

a) There is strong evidence that mu = 100
b) There is strong evidence that mu does not equal 100
c) There is strong evidence that mu > 100
d) There is strong evidence that mu < 100
e) If mu were equal to 100, it would be unusual to obtain data such as those observed

Multiple Choice Question example

- The P-value for testing the null hypothesis mu=100 (two-sided) is P-value=.001. Suppose that in addition you know that the z score of the test statistic was z=3.29. Then

a) There is strong evidence that mu = 100

b) There is strong evidence that mu > 100

c) There is strong evidence that mu < 100

- If the conclusion of a testing hypothesis result in “reject the null hypothesis”. Suppose the alpha level used is 0.01.

- What would be the conclusion if we set the alpha = 0.05 instead? (everything else remain the same)

Attendance Survey Question 25

- On a 4”x6” index card
  - Please write down your name and section number
  - Today’s Question:
  - If we change the alpha level from 0.05 to 0.01, then the previous conclusion of “reject the null hypothesis”

a. may change   b . Remains un-changed

Multiple Choice Question exampleI

- A 95% confidence interval for mu is (96,110). Which of the following statements about significance tests for the same data are correct?
  a) In testing the null hypothesis mu=100 (two-sided), P>0.05
  b) In testing the null hypothesis mu=100 (two-sided), P<0.05
  c) In testing the null hypothesis mu=x (two-sided), P>0.05 if x is any of the numbers inside the confidence interval
  d) In testing the null hypothesis mu=x (two-sided), P<0.05 if x is any of the numbers outside the confidence interval