STA 291
Lecture 7

- Probability [rules]
- Working with events

Review
Probability: Basic Terminology

- **Sample Space: [** denoted by S **]** The collection of all possible outcomes of an experiment.
- **Event: [** denoted by A, B, C, etc **]** a specific collection of outcomes.
- **Simple Event:** An event consisting of just one outcome.

- Rule 1: $0 \le P(A) \le 1$
- Rule 2: P(S) =1
- Rule 3: $P(A) = 1 - P(A^c)$

Assigning Probabilities to Events

- The probability of an event is nothing more than a value between 0 and 1.   In particular:

--- 0 implies that the event will not occur

--- 1 implies that the event will occur for sure

- Never have probability > 1, never < 0 .

Equally Likely Approach

- Examples:

1. A deck of 52 cards, well shuffled. Pick one. Let event A={ace, any suits}, P(A) =

2. Roll a fair die
   – The probability of the event “4 or above” is

- Roll a fair die 2 times: there are 6x6=36 possible outcomes. Each one has 1/36 probability

Counting method

- Suppose at every step, you always have **k** choices, and there are **m** steps

- Total number of choices = k x k x k …k = k to m power

- Example: pick 3 lotto [10 to 3 power]
- Roll a die 3 times [6 to 3 power]

Example: using rule 3

- Flip a fair coin 7 times
- A = { at least one head }
- P(A) =1 – 1/128 = 127/128

More rules

- Rule for union/sum (a general rule and a simplified rule)

- Rule for intersection/product (a general rule and a simplified rule)

Union and Intersection of events

- Let *A* and *B* denote two events.
- **The union of two events:** All the outcomes in *S* that belong to at least one of *A* or *B or both*. The union of *A, B* is denoted by $A \cup B$
- **The intersection of two events:** All the outcomes in *S* that belong to both *A* and *B*. The intersection of *A* and *B* is denoted by

  $A \cap B$

- A Union B = pink, blue and purple
- A intersect B = purple

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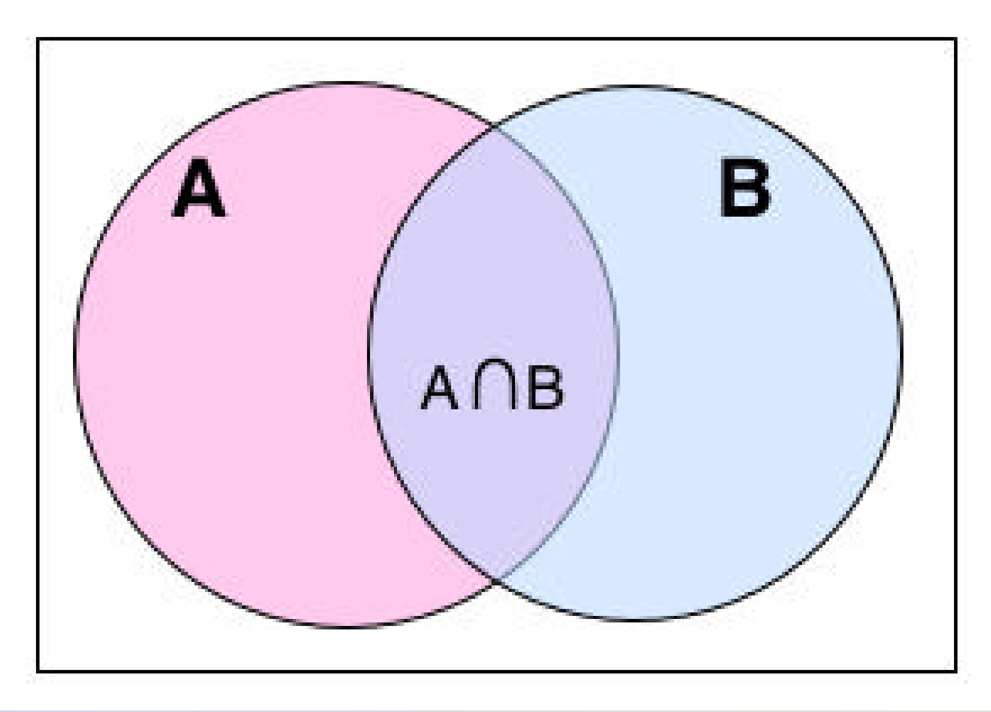
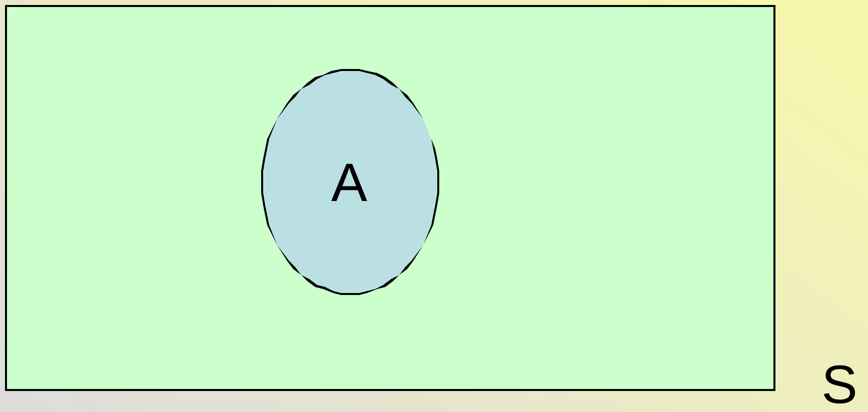


Complement

- Let *A* denote an event.
- **The complement of an event** ***A*****:** All the outcomes in the sample space *S* that **do not belong** to the event *A*. The complement of *A* is denoted by A^C = green

A = blue



Law of Complements:

$$P(A^C) = 1 - P(A)$$

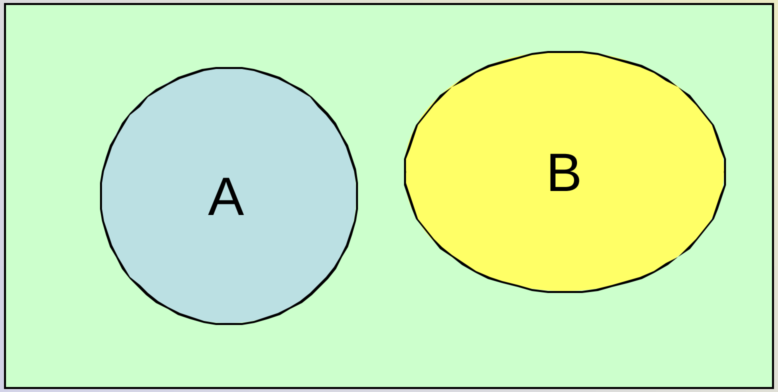
- **Example:** If the probability of getting a "working" computer is 0.85,
- What is the probability of getting a defective computer?
- **Example** if the probability of hitting the

free throw is 0.68, then what is the probability of missing?

Disjoint Events

- Let *A* and *B* denote two events.
- **Disjoint (or mutually exclusive) events:** *A* and *B* are said to be disjoint if there are no outcomes common to both *A* and *B*.
- Using notation, this is written as $A \cap B = \varnothing$
- Note: The last symbol, $\varnothing$ denotes the null set or the empty set.

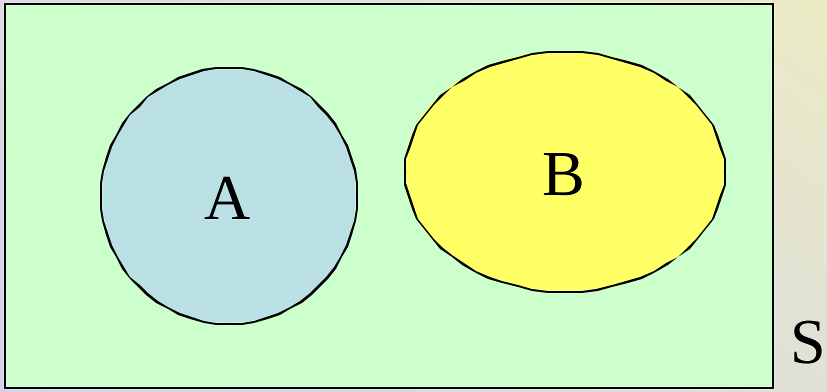
- A and B are disjoint,
- A and B are mutually exclusive, no overlap



Probability of Disjoint Events

Let A and B be two events in a sample space S. The probability of the union of two disjoint (mutually exclusive) events A and B is

$$P(A \cup B) = P(A) + P(B).$$



- Deck of 52 cards
- Let A = get an Ace
- Let B = get a Queen

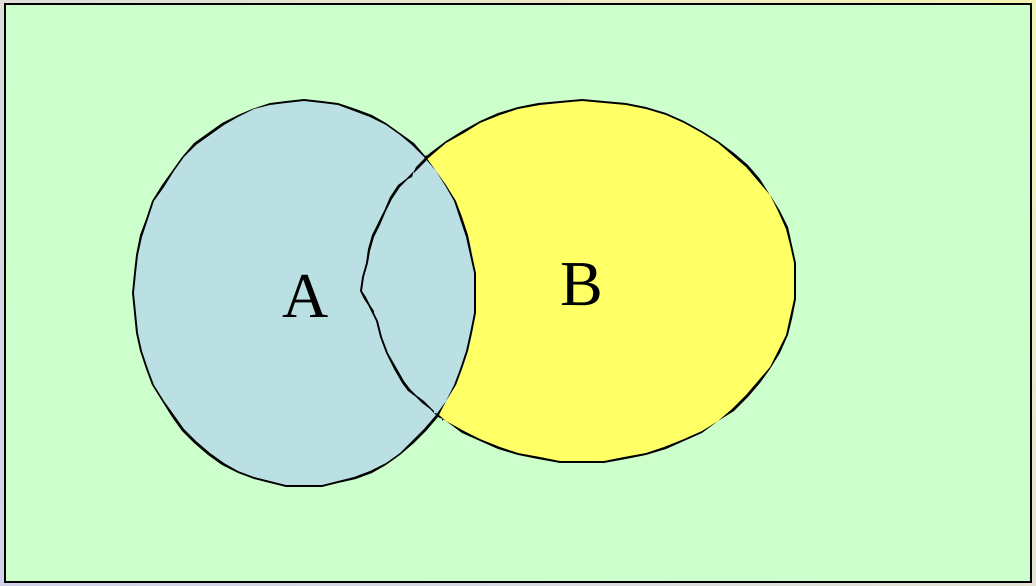
- Then A , B are disjoint. Therefore P(A or B) = P(A) + P(B)

- If C = get a spade. Is A , C disjoint?

Additive Law of Probability

Let A and B be two events in a sample space S. The probability of the union of A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$



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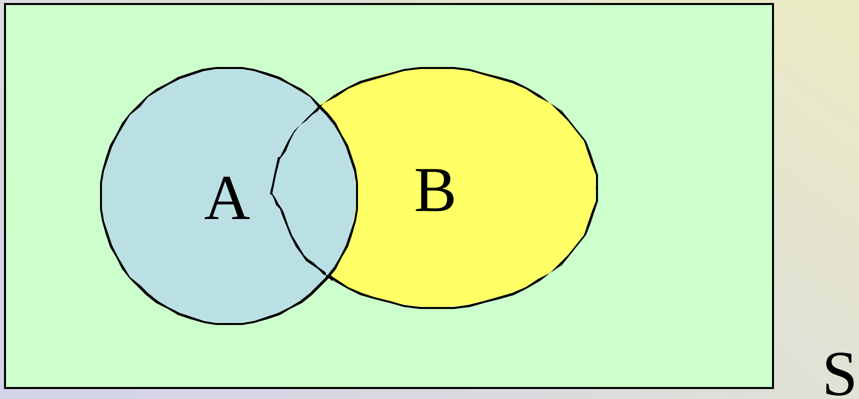
Using Additive Law of Probability

Example: At State U, all first-year students must take chemistry and math. Suppose 15% fail chemistry, 12% fail math, and 5% fail both.

Suppose a first-year student is selected at random. What is the probability that student selected failed at least one of the courses?

What is the probability that he/she pass both?



Rules for intersection

- (A shortcut for independent events)

- If two events A, B are `independent’ then

$$P(A \cap B) = P(A)P(B).$$

Independent

- How do we know if events are independent?

- If A and B are not independent, then

$$P(A \cap B) = P(A)P(B \mid A).$$

- Flip a *fair* coin two times
- Sample space = { HH, HT, TH, TT }
- Using equal likely probability assignment

- A = { exactly one H }

- P(A) =

- Cannot use this for biased coins

- Outcomes, and their probabilities in a sample space may be given in a contingency table. (r x c table)

- Example

- Last digit of your Social Security number

[most likely random and equally likely to be 0, 1, 2, …, 9]

Attendance Survey Question 7

- On a 4”x6” index card
  - Please write down your name and section number
  - Today’s Question:

Your prediction of Super Bowl this Sunday

___Indianapolis

___New Orleans