where we have used the identity

$$1 = \left(\frac{1}{2} + \frac{1}{2}\right)^M = \sum_{j=0}^{M} \binom{M}{j} \left(\frac{1}{2}\right)^M$$

Hence, from Equation (4.24)

$$\pi_i = \binom{M}{i} \left(\frac{1}{2}\right)^M, \quad i = 0, 1, \ldots, M$$

Because the preceding are just the binomial probabilities, it follows that in the long run, the positions of each of the $M$ balls are independent and each one is equally likely to be in either urn. This, however, is quite intuitive, for if we focus on any one ball, it becomes quite clear that its position will be independent of the positions of the other balls (since no matter where the other $M-1$ balls are, the ball under consideration at each stage will be moved with probability $1/M$) and by symmetry, it is equally likely to be in either urn.

**Example 4.36** Consider an arbitrary connected graph (see Section 3.6 for definitions) having a number $w_{ij}$ associated with arc $(i, j)$ for each arc. One instance of such a graph is given by Figure 4.1. Now consider a particle moving from node to node in this manner: If at any time the particle resides at node $i$, then it will next move to node $j$ with probability $P_{ij}$ where

$$P_{ij} = \frac{w_{ij}}{\sum_j w_{ij}}$$

and where $w_{ij}$ is 0 if $(i, j)$ is not an arc. For instance, for the graph of Figure 4.1, $P_{12} = 3/(3+1+2) = \frac{1}{2}$.

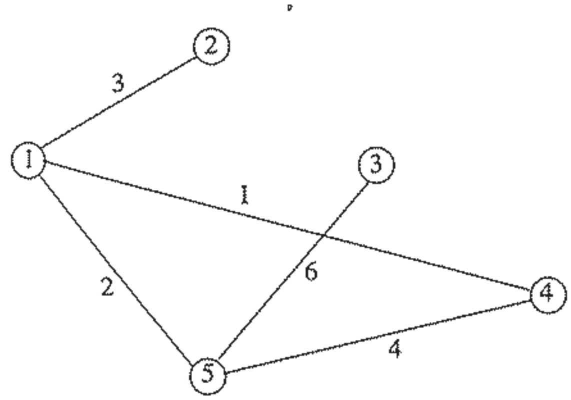


Figure 4.1 A connected graph with arc weights.