

1. problem 23 (c) on page 278 of Ross book (10th Ed).  
Find the long-run average number of storms per year.

The transition probability matrix for the start is:

$$P^{(0)} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

Solving the following system of equation will yeild the proper long run probabilities of being in each state:

$$\begin{aligned} \pi_0 &= \frac{1}{2}\pi_0 + \frac{1}{3}\pi_1 \\ \pi_1 &= \frac{1}{2}\pi_0 + \frac{2}{3}\pi_1 \\ 1 &= \pi_0 + \pi_1 \end{aligned}$$

Which has solutions:

$$\begin{aligned} \pi_0 &= \frac{2}{5} \\ \pi_1 &= \frac{3}{5} \end{aligned}$$

Which means that we expect in the long run to have 40% of the years be good and about 60% of the years to be bad. So the expected number of storms per year is:

$$\text{Expected number} = .4 \times 1 + .6 \times 3 = 2.2$$

So the expected number of storms per year is 2.2.