2. Write $N(t) = N(t) - N(s) + N(s)$; $N(r) = N(r) - N(s) + N(s)$

We have

$$\mathbb{E}\left[N(t)N(r) \mid N(s) \right] = \mathbb{E}\left[(N(t) - N(s) + N(s))(N(r) - N(s) + N(s)) \mid N(s) \right]$$

$$= \mathbb{E}\left[(N(t)-N(s))(N(r)-N(s)) \mid N(s) \right] + \mathbb{E}\left[N^2(s) \mid N(s) \right]$$

$$+ \mathbb{E}\left[(N(t) - N(s))N(s) \mid N(s) \right] + \mathbb{E}\left[(N(r)-N(s))N(s) \mid N(s) \right]$$

$= \mathbb{E}\,(N(t)-N(s))(N(r)-N(s))$ ..... Since both brackets are indep. of $N(s)$.

$+ \; N^2(s)$ ..... when given $N(s)$; $N(s)$ is like a const.

$+ \; N(s) \cdot \mathbb{E}\,(N(t)-N(s))$ ..... use both reasons above.

$+ \; N(s)\, \mathbb{E}\,(N(r)-N(s))$ ditto.

$= \mathbb{E}\, N(t-s)N(r-s)$ .... by stationary [starting at s vs starting at 0]

$+ \; N^2(s)$

$+ \; N(s) \cdot \mathbb{E}\, N(t-s)$ ... stationary

$+ \; N(s) \cdot \mathbb{E}\, N(r-s)$ ... stationary

$= N^2(s) + N(s)\cdot \lambda(t-s) + N(s)\cdot\lambda(r-s)$

$+ \; \lambda(t-s) + \lambda(t-s)\cdot\lambda(r-s)$ ※

↳ see note on right. →

from HW #1 problem 3 (a)

$Cov(N(u), N(v)) = \lambda \cdot v$, $0 < v < u$.

$\Rightarrow \mathbb{E}\, N(u)N(v) = \lambda v + \lambda u \cdot \lambda v$

here $v = t-s$

$u = r-s$. $0 < t-s < r-s$