Notes for Sta 624 Stochastic Processes, Spring 2012

The page number etc below is based on 10th ed. of the book. We shall update them as we go along.

Some topics we covered.

Exponential random variables.

Memoryless property, constant hazard.
Minimum of independent exponential r.v.s is still an exponential.
prob(exp1 > exp2) etc.

Poisson process.

Three equivalent definitions
mean, intensity, and distribution
stationary and independent increments
splitting, combining/merging Poisson processes
arrival times, inter-arrival times
First, try to go through all the examples in the chapter 5.
If you want some more practice problems try problems at the end of chapter 5
like #26, #28, #30, #38, #39, #53, #58, #62

Some Topics about Markov Chain (MC).

P the transition probability matrix.
P^n the n-step transition probability matrix.
States that communicates with each other form a class.
Recurrent/transient of a state, of a class (how to identify?)
A MC with one class is irreducible

For recurrent states:

The stationary distribution π of an irreducible, recurrent MC satisfy

$$\pi = \pi P,$$

and π is a probability.

The continuous state version (time is still discrete) of this is

$$\int p(x|y)f(y)dy = f(x)$$

and $f(x)$ is a density. (no more than this and the example of note2)

Mean time between visits to a recurrent state i (start in i); mean recurrence time $= 1/\pi_i = m_{ii}$.

The expected number of transitions (mean time) it take to visit state j , starting from state i is denoted by m_{ij}. [we did not give a formula, but may use the trick of change j to absorbing state, and find the absorption time.]

For transient states:

Mean number of visits to a transient state j , starting from a transient state i (in the same class) before the MC got absorbed is s_{ij}, can be obtained from the matrix $S = (I - P_t)^{-1}$.

(on page 228) $f_{ii} = f_i$ (on page 191, 192), probability of ever return to state i when starting from i.

(remember for transient states, $f_i < 1$, for recurrent, $f_i = 1$.)

The mean number of visits to state i (starting in i) is $1/(1 - f_i)$, (by the expectation of the geometric distribution).

Therefore we have $1/(1 - f_i) = s_{ii}$. We also have $s_{ii} = 1 + \sum_{n=1}^{\infty} P_{ii}^n$.

For off diagonal values of f_{ij}, we have $s_{ij} = f_{ij}s_{jj}$. From here you can solve for f_{ij}, where f_{ij} is the probability of visiting state j (starting in i).

Important Examples

Random walk, or Gambler's ruin problem. (reflection/absorption boundary, finite/infinite states, symmetric or not, dimension 1 or 2).

Random walk on a connected graph, or connected rooms.

Ehrenfest model

Continuous State Space MCMC. Metropollis algorithm.

Transition kernel, Detailed Balance equation.

Bayesian example.

Brownian motions. Geometric Brownian Motions.

Definition and distributional assumption about $B(t)$.

Definition of Martingales. Example of martingales. (those related to Brownian Motion and those related to Poisson process, and those related to the partial sum of independent random variables (in particular random walk))

First hitting time for Brownian motion.

Optional stopping theorem for martingales (no proof) and its applications.