Modeling of Right Censored Data

What we observe are lifetimes that have been subject to censoring already:

T = observed time [either the true lifetime or censored lifetime]

δ = indicator if the associated T is a true lifetime ($\delta = 1$) or censored lifetime ($\delta = 0$).

We assume there are X and C, represent the true lifetime and associated follow-up time, and

$$T = \min(X, C) \; ; \quad \delta = I[X \le C] .$$

We observe T, δ. But we wish to estimate the CDF of X.

Assume $X \perp\!\!\!\perp C$. [Independent Censoring]

$T = \min(X, C)$; $\delta = I[X \leq C]$.

Let

$$U_1(t) = P(T \leq t, \delta = 1); \quad V_1(t) = P(T > t, \delta = 1)$$

$$U_0(t) = P(T \leq t, \delta = 0); \quad V_0(t) = P(T > t, \delta = 0).$$

Notice

$$(T \leq t, \delta = 1) \iff (T \leq t, X \leq C)$$

OR $(T = s, \delta = 1) \iff (T = s, X \leq C) \iff (X = s, X \leq C)$

$$\iff (X = s, s \leq C)$$

So, $P(T = s, \delta = 1) = P(X = s, s \leq C) \underset{\uparrow}{=} P(X = s) P(C \geq s)$

use indep $X \perp\!\!\!\perp C$

$$V_1(t) = \int_{s=t}^{s=+\infty} P(T = s, \delta = 1) = \int_t^{\infty} P(C \geq s) P(X = s)$$

$$= \int_t^{\infty} [1 - F_c(s)] \, dF_x(s) \quad \text{----} \quad (1)$$

Similarly

$$V_0(t) = \int_{s=t}^{s=+\infty} P(T = s, \delta = 0) = \int_t^{\infty} [1 - F_x(s)] \, dF_c(s) \quad \text{---} \quad (2)$$

i.e. [use $F_c(\cdot)$ & $F_x(\cdot)$ you can determine $V_0(\cdot)$, $V_1(\cdot)$]

For the other direction, notice

$$V_1(t) + V_0(t) = \int_t^{\infty} [1-F_x(s)]\,dF_c(s) + [1-F_c(s)]\,dF_x(s)$$

$$= -\int_t^{\infty} d[1-F_x(s)][1-F_c(s)] = [1-F_x(t)][1-F_c(t)],$$

and

$$dV_1(s) = -[1-F_c(s)]\,dF_x(s)$$

$$dU_1(s) = [1-F_c(s)]\,dF_x(s)$$

Finally

$$\frac{dV_1(s)}{V_1(s)+V_0(s)} = \frac{-\cancel{[1-F_c(s)]}\,dF_x(s)}{[1-F_x(s)]\,\cancel{[1-F_c(s)]}}$$

$$= \frac{-dF_x(s)}{1-F_x(s)} \qquad \cdots\cdots (3)$$

Integration on both sides.

$$\int_0^t \frac{dV_1(s)}{V_1(s)+V_0(s)} = \int_0^t \frac{-dF_x(s)}{1-F_x(s)} \doteq \log[1-F_x(t)]$$

only for Cont. $F_x(\cdot)$

Recall

[$V_1(\cdot)$ and $V_0(\cdot)$ Can be estimated from Censored data easily]