

TSP Heuristics: Domination Analysis and Complexity

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Abstract

We show that the 2-Opt and 3-Opt heuristics for the traveling salesman problem (TSP) on a complete graph K_n produce a solution no worse than the average cost of a tour in K_n in a polynomial number of iterations. As a consequence, the domination numbers of the 2-Opt and 3-Opt, Carlier-Villon, Shortest Path Ejection Chain, and Lin-Kernighan heuristics are all at least $\frac{(n-2)!}{2}$. The domination number of the Christofides heuristic is shown to be no more than $\lceil \frac{n}{2} \rceil!$, and for the Double Tree heuristic and a variation of the Christofides heuristic the domination number is shown to be one (even if the edge costs satisfy the triangle inequality). Further, unless $P=NP$, no polynomial time approximation algorithm exists for the TSP with domination number at least $(n-1)! - k$ for any constant k or with domination number at least $(n-1)! - (\frac{k}{k+1}(n+r))! - 1$ for any constant r and any constant k such that $k \equiv -1 \pmod{n+r}$. The complexity of finding the value of the median tour and of similar problems is also studied.

Key words: Domination Analysis, Approximation Algorithms, Traveling Salesman Problem, Computational Complexity

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