

Asymptotically Optimal Lower Bounds
For the Condition Number of a Real Vandermonde Matrix¹

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ABSTRACT

Lower bounds on the condition number $\min \kappa_p(V)$ of a Vandermonde matrix V are established in terms of the dimension n or n and the largest absolute value among all nodes that define the Vandermonde matrix. Many bounds here are asymptotically sharp, and compare favorably to those of the same kind due to Gautschi and Inglese (*Numer. Math.*, 52 (1988), 241–250) who considered either all positive nodes or real symmetric nodes and those due to Tyrtyshnikov (*Numer. Math.*, 67 (1994), 261–269). As consequences our bounds imply that $\min \kappa_p(V)$ over all possible real nodes goes to ∞ as fast as $(1 + \sqrt{2})^n$, and $\min \kappa_p(V)$ over all possible nonnegative nodes goes to ∞ as fast as $(1 + \sqrt{2})^{2n}$. Extensions were made to confluent Vandermonde matrices and rectangular Vandermonde matrices, including brief outlines on how to deal with Vandermonde-like matrices and complex Vandermonde matrices.

¹This report is available on the web at <http://www.ms.uky.edu/~math/MAreport/PDF/04-05.pdf>.

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