Calculus II — *Spring 2004*

MA114:004-006 — *Russell Brown*

1. Memorize items 1–17 on the integral table in the textbook. Memorize is defined to mean 1. know the formula 2. know why the formula is true.

2. Know the trigonometric identities as discussed on the trigonometry handout for the first examination.

3. Integration by parts and its use in evaluating integrals.

4. Proving reduction formulae.

5. Evaluating integrals involving non-negative powers of $\sin x$ and $\cos x$.

6. Evaluating integrals of the form

$$\int x^j (a^t - t^2)^{k/2}\, dt.$$

   I will not set examination questions over the other trigonometric substitutions.

7. Using the division algorithm to rewrite an improper rational function as the sum of a polynomial and a proper rational function. (Review topic from algebra.)

8. Write out the form of the partial fractions decomposition and finding the constants in partial fractions decompositions.

9. Integrating the rational functions which arise in a partial fractions decomposition, except integrals of the form $\int 1/(Q(t))^j\, dt$ where $j > 1$ and $Q(t)$ is an irreducible quadratic.

10. The substitution $\sqrt[n]{ax+b}$.

11. Approximating integrals by the trapezoid rule and Simpson's rule.

12. Using the error estimates to compute integrals to within a specified absolute error.

13. Consider improper integrals at infinity.

14. Find areas as improper integrals.

15. The comparison theorem.

16. Solve initial value problems for separable differential equations.

17. Solve mixing problems where the volume is contant.

Sample problems

1. Evaluate the following integrals

   (a) $\int x^2 \sin(2x)\,dx$

   (b) $\int e^{2x} \cos(3x)\,dx$

   (c) $\int (\ln x)^2\,dx$

   (d) $\int \sec^2 x \tan x\,dx$

   (e) $\int \sin^3 x \cos^3 x\,dx$

   (f) $\int \sin^2 x\,dx$

   (g) $\int \sin^2 x \cos^4 x\, m dx$

   (h) $\int \tan x\,dx$

   (i) $\int \frac{1}{\sqrt{4^2 - x^2}}\,dx$

   (j) $\int \frac{x}{\sqrt{4^2 - x^2}}\,dx$

   (k) $\int (2x+1)^3\,dx$

   (l) $\int \frac{x}{x^2+4x+8}\,dx$

   (m) $\int \frac{1}{x^3+x^2}\,dx$

   (n) $\int \frac{1}{x^3+x}\,dx$

   (o) $\int \frac{x^2}{x+1}\,dx$

   (p) $\int_0^1 \frac{x}{1+\sqrt{x}}$

2. Use integration by parts to establish an equation relating

$$\int_0^1 x^{100} e^x\,dx \quad \text{and} \int_0^1 x^{99}\,dx.$$

3. Give the form of the partial fractions decomposition for the following functions. Do not solve for the constants.

$$\frac{x^2}{(x^2 - 3x - 4)^2}, \quad \frac{x^3}{x^4 - 1}, \quad \frac{x}{x^4 + x^2}.$$

4. Define the following terms. Give an example of each one.

   (a) Rational function.

   (b) Proper rational function

   (c) Irreducible quadratic polynomial

5. Use Simpson's rule with $n = 6$ to approximate the integral

$$\int_2^5 \sin(0.5x)\, dx$$

6. Use the trapezoid rule with $n = 7$ to approximate the integral

$$\int_2^5 \sin(0.5x)\, dx$$

7. Explain how to use Simpson's rule to approximate $\ln 2$ to an error of at most 10^{-3}. (You do not need to memorize the error rule, this will be given to you on the exam.)

8. Explain how to use the trapezoid rule to approximate $\int_0^4 \sin(3x)\, dx$ to an error of at most 10^{-3}. (You do not need to memorize the error rule, this will be given to you on the exam.)

9. State the comparison theorem for improper integrals.

10. Find the integral $\int_0^\infty xe^{-x}\, dx$.

11. Find the area between the curve $x = e^y$, $y - axis$ and the line $y = 2$. Since this region is infinitely long, it must have infinite area, right?

12. Use the comparison theorem to determine if the following integrals converge or diverge.

$$\int_0^\infty \frac{e^{-x}}{2 + \sin x}\, dx \qquad \int_0^\infty \frac{1}{3 + 2x + \cos x}\, dx$$

13. Solve the differential equation

$$y' = 4 - y^2, \quad y(0) = 1.$$

Find

$$\lim_{t \to \infty} y(t).$$

14. Suppose that a tank initially contains 30 grams of salt dissolved in 400 liters of water. Brine with a concentration of 4 grams/liter of salt flows in at a rate of 3 liters/minute. The tank is perfectly mixed.

   (a) Find the mass $M(t)$ of salt in the tank after t minutes.
   (b) Find the amount of salt in the tank after 30 minutes.
   (c) Find $\lim_{t \to \infty} M(t)$.