Calculus II — *Spring 2004*
MA114:004-006 — *Russell Brown*

The third hour exam will be on Tuesday, 13 April 2004 from 7:30-9:30pm in CB 110. The final exam will be from 10:30am-12:30pm in CP 320 on Wednesday, 5 May 2004.

1. Memorize items 1–17 on the integral table in the textbook. Memorize is defined to mean 1. know the formula 2. know why the formula is true.
2. Know the trigonometric identities as discussed on the trigonometry handout for the first examination.
3. Compute lengths of curves which are presented as graphs.
4. Find limits of sequences.
5. Definition of monotone bounded sequences. Finding limits of bounded monotone sequence.
6. Definition of convergent and divergent series.
7. Geometric series and telescoping series.
8. Simple test for divergence.
9. The integral test.
10. Using the integral test to test convergence of p-series.
11. Using the integral test to estimate series.
12. The comparison test for positive series.
13. Using the limit comparison test to test for convergence or divergence of positive series.
14. Use the alternating series test to test convergence.
15. Use the alternating series estimation theorem to approximate the value of a series.
16. Absolutely and condittionally convergent series.
17. Using the ratio test to determine convergence.
18. Find interval and radius of convergence for power series using ratio test.
19. Termwise differentiation and integration of power series.
20. Find power series for functions such as $1/(1+x^4)$, $\ln x$, $\tan^{-1} x$ and $1/(1+x)^2$.

Sample problems and review problems.

1. Find the length of the graph of $f(x) = \ln \cos x$ for $0 \le x \le \pi/4$.

2. Find the limits:
$$\lim_{n\to\infty} \frac{\sin n}{n} \qquad \lim_{n\to\infty} \frac{n^2}{n-1} \qquad \lim_{n\to\infty} \cos(\pi n)$$

3. Find a simple form for the sum
$$r^{-1} + 1 + r + r^2 + r^3 + \ldots + r^{100}.$$
Explain every detail. Do not use the formula from the text.

4. Write the repeating decimal as a fraction.
$$0.\overline{37}$$

5. Determine if each of the following series converge and if the series converges, find the value of the series.
$$\sum_{n=1}^{\infty} (3 \cdot 2^{-n} + 2 \cdot 3^{-n}).$$

6. Find the value of the series
$$\sum_{n=2}^{\infty} \frac{1}{n^2-1}.$$

7. Determine if the series converges. Explain your answer.
$$\sum_{n=2}^{\infty} 3^n.$$

8. One of the following statements is false. Which one is it? Give an example which shows that the statement you chose is false.
If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.
If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

9. State the integral test. Draw your favorite picture.

10. Does the series $\sum_{n=1}^{\infty} \frac{n}{n^2+16}$ converge? Explain.

11. Does the series $\sum_{n=1}^{\infty} ne^{-n}$ converge? Explain.

12. Find N so that the difference between $\sum_{n=1}^{N} \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is at most 10^{-6}.

13. Define *convergent series, divergent series, absolutely convergent series and conditionally convergent series.*

14. Can you find a series which is absolutely convergent, but not convergent?

15. Determine if the following series are conditionally convergent, absolutely convergent or divergent. Justify your answer.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1} \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2+1} \quad \sum_{n=1}^{\infty} n!2^{-n} \quad \sum_{n=1}^{\infty} \frac{n!}{(n+2)!}.$$

$$\sum_{n=1}^{\infty} \sin(1/n^2) \quad \sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2} \quad \sum_{n=1}^{\infty} \frac{n^3-n^2}{n^{23}+12n^{13}}$$

16. Give an example to show that we can have

$$\left|\sum_{n=1}^{\infty} a_n - \sum_{n=1}^{N} a_n\right| > a_{N+1}.$$

17. State the ratio test.

18. If a series $\sum_{n=1}^{\infty} a_n$ converges, is it true that

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1?$$

19. Can you find a series, $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0.5$ and the series converges conditionally?

20. Can you find a series, $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ the series diverges?

21. Determine the interval of convergence:

$$\sum_{n=1}^{\infty} \frac{x^n}{n!} \quad \sum_{n=1}^{\infty} \frac{(2x+1)^n}{n} \quad \sum_{n=1}^{\infty} x^{n^2} n!$$

22. (a) Find a power series centered at 0 for the function

$$F(x) = \int_0^x \frac{1}{1+t^3}\, dt.$$

(b) Find an approximate value for $F(0.3)$ that is within 10^{-4} of the exact value. Prove that your answer is correct. You may check your answer by using your calculator to evaluate the integral.

23. Find the power series centered at 0 for $\ln(1-x)$.

24. Find the power series centered at 0 for $1/(1+x^2)^2$.