The Earliest Solution of the Biquadratic.

THE oldest complete solution of the biquadratic is attributed to Ferrari (1522–65). He is supposed to have solved the equation $x^4 + 6x^2 - 60x + 36 = 0$, proposed as a challenge to mathematicians of his time. The historians[1] of mathematics have been for some time familiar with a solution of a biquadratic by Bhaskara (1150 A.D.), an Indian mathematician. But the principle of his method hidden away in a numerical example deserves to be made explicit. Bhaskara solves

$$x^4 - 2x^2 - 400x = 9999$$

by adding to both sides $4x^2 + 400x + 1$ so as to make them perfect squares. This method can be easily generalised thus:

Suppose $x^4 + qx^2 + rx = s$.

Add to both sides $ax^2 - rx + b$ and choose a, b so that both sides may be perfect squares.

Then, we have

$$x^4 + (a + q)\, x^2 + b = ax^2 - rx + (s + b)$$

and the conditions to be satisfied by a, b are

$$(a + q)^2 = 4b, \quad r^2 = 4a\,(s + b).$$

Bhaskara has guessed a and b; but we may eliminate b and get a as a root of the *cubic*

$$a^3 + 2a^2q + a\,(q^2 + 4s) - r^2 = 0$$

exactly the same as the one due to Descartes (1637). The roots of the biquadratic are obtained by solving

$$x^2 + \frac{a + q}{2} = \pm \sqrt{a}\left(x - \frac{r}{2a}\right).$$

So far as we know, Bhaskara's is the earliest attempt at the solution of the biquadratic and is in line with the later solutions of Ferrari, Vieta and Descartes, though, of course, the *cubic* was not there. At a period when even the negative root was admitted with great hesitancy, it is no wonder that imaginary roots should have been regarded as spurious and unfit to mix with the other numbers. Bhaskara therefore naturally recognised only the real positive root of his biquadratic and did not think of the others.

A. A. KRISHNASWAMY AYYANGAR.

Maharaja's College,
Mysore,
September 12, 1938.

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[1] *History of Hindu Mathematics*, Part II, by B. Datta and A. N. Singh. (Motilal Banarsi Das, Lahore, 1938.)

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