

Dissecting the Cube

Lesson Plan

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Goal: The goal of this lesson is to gain an intuitive grasp of the volume of a pyramid (and more generally a cone).

Grade and Course: 8th grade, pre-algebra

KY Standards: MA-08-3.1.3, MA-08-2.1.4 (Geometry: Shapes & Relationships)

Objectives: This lesson is intended to boost students' conceptual understanding of the volume of common three-dimensional solids. In particular, we seek to justify (not rigorously, though) the fact that the volume $V = \frac{lw h}{3}$ of a pyramid height h with a rectangular base of dimension $l \times w$ is exactly $1/3$ the volume of the box with the corresponding dimensions $l \times w \times h$. This result can be used to extrapolate the volume of any three-dimensional cone.

Resources/materials needed: worksheets, handouts (with cutouts), scissors, and tape

Description of Plan: The lesson begins with a review of the volumes of boxes and cylinders (the volume of prisms may also be reviewed here). We then pose the problem of determining the volume of a pyramid. We constructively establish the volume by making three congruent right pyramids – each with equal length, width, and height – and observe that we can form a cube out of these three congruent ‘solids’. From this observation we derive the formula for the volume of the right pyramid as $1/3$ of the volume of the box. We observe that this formula generalizes in the natural way to arbitrary pyramids with triangular bases. In fact, it easily generalizes to pyramids with arbitrarily shaped bases. These solids are more commonly referred to as cones. For the case of a cone with a circular base, one can check that its volume is indeed $1/3$ of the volume of the corresponding prism (a cylinder) with the same base and height using [Relational GeoSolids](#).

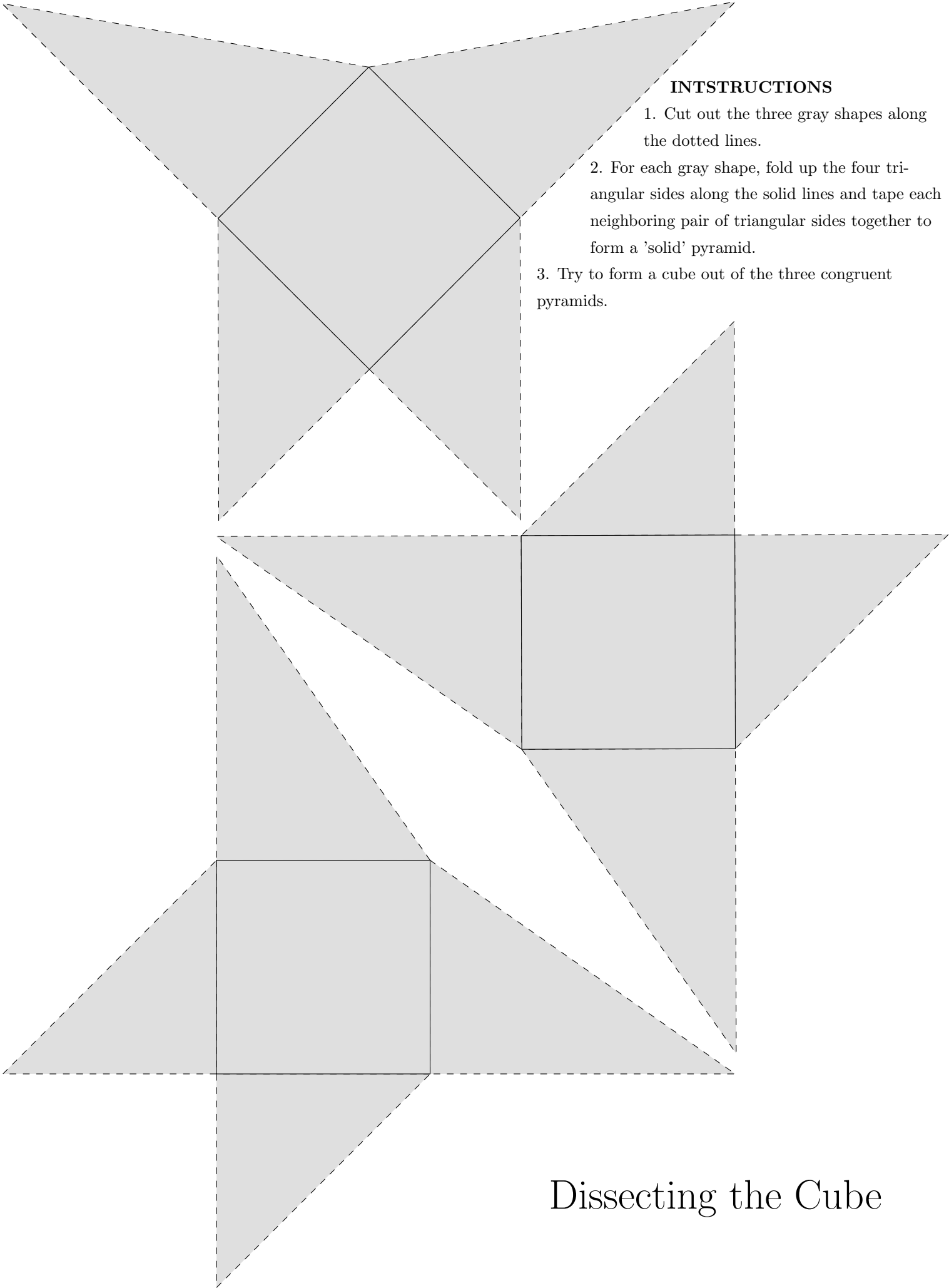
Lesson Source: original

Instructional Mode: individual

Date Given: April 21, 2008

Estimated Time: 35 minutes

Date Submitted to Algebra³: June 25, 2008



INTSTRUCTIONS

1. Cut out the three gray shapes along the dotted lines.
2. For each gray shape, fold up the four triangular sides along the solid lines and tape each neighboring pair of triangular sides together to form a 'solid' pyramid.
3. Try to form a cube out of the three congruent pyramids.

Dissecting the Cube

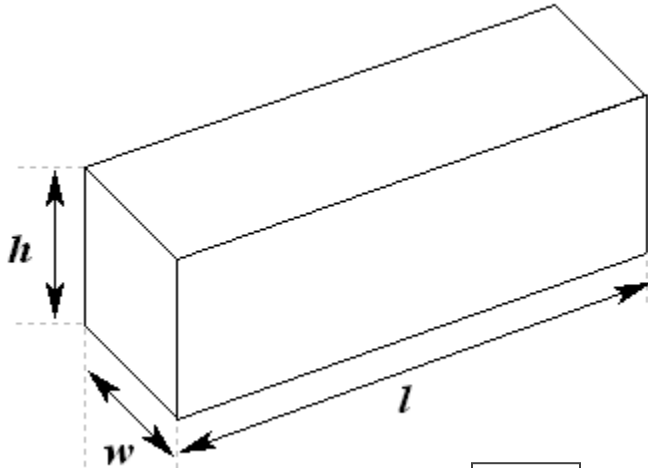
Name: _____

Period: _____

Volumes of Pyramids

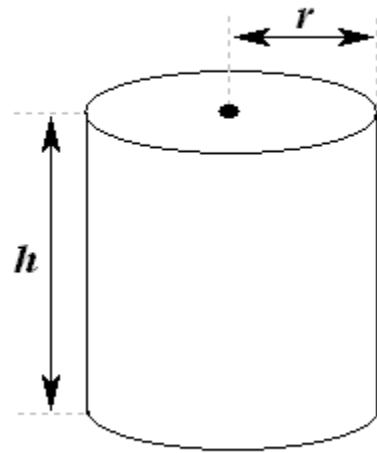
1. Recall the volumes of these basic solids:

Rectangular Solid:



Volume =

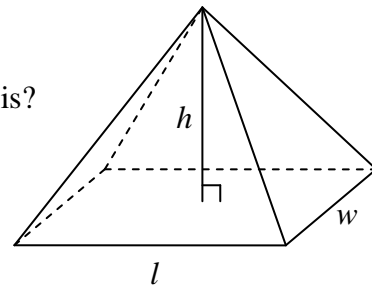
Cylinder:



Volume =

Let's try to see how the volumes of these shapes relate to the volumes of pyramids and cones.

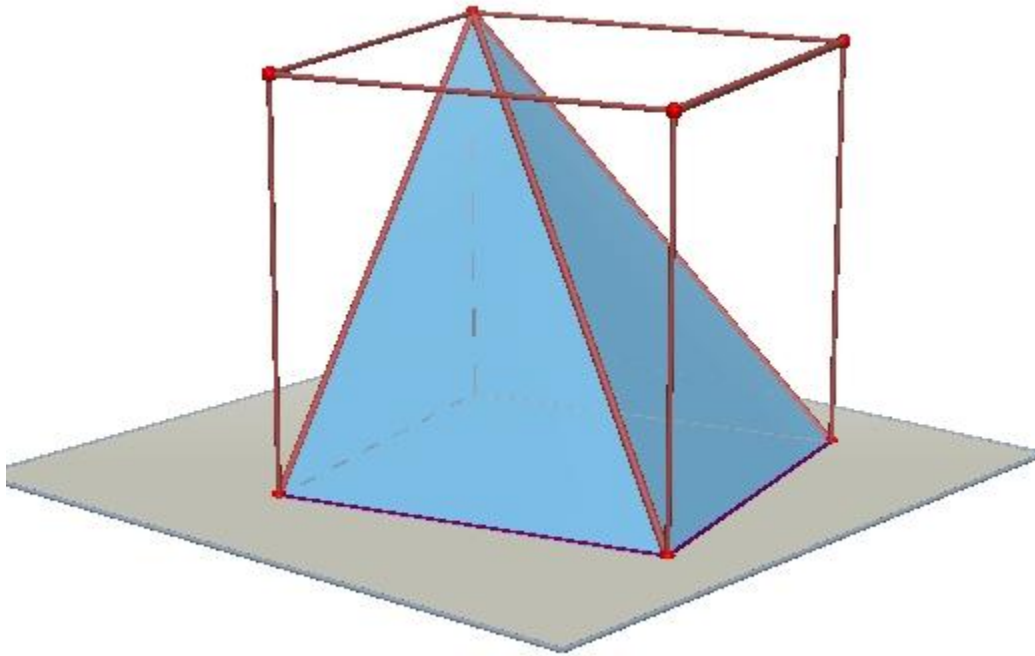
2. Do you know what the volume of this pyramid is?



Follow the instructions on the "Dissecting the Cube" handout with the gray shapes.

3. What relationship is there between the pyramids you constructed and the cube whose side length equals the side length of the base of the pyramid?
4. Based on this relationship, what can you deduce about the relationship between the volume of the cube and the volume of one of the pyramids?

5. Write a formula expressing the volume V_{pyr} of one of the pyramids you made in terms of the volume V_{cube} of the cube that the three of them formed.



6. Make a conjecture (if you don't already know) as to what the volume V_{pyr} of the pyramid in 2. is in terms of the volume of the box of dimension $l \times w \times h$.

Check with your instructor to learn if the conjecture you made is correct.