An Introduction to Measure Theory

Lesson Plan

Cube Fellow: Chris Mattingly   Teacher Mentor: Lisa Osborne

Goal: To communicate the basic ideas of the field I am researching in to my students.

Grade and Course: 9th grade – Algebra I

KY Standards:

Objectives: By the end of the lesson, students should understand what a measure does and why 1-, 2-, and 3-dimensional objects have their dimensions.

Resources / Materials needed: None

Description of the plan: We will go through a set of notes where the students first think about what they already know about dimension and measuring, and then discuss some of the basic properties. It will be a discussion format followed by a short lecture.

Lesson source: Original lesson

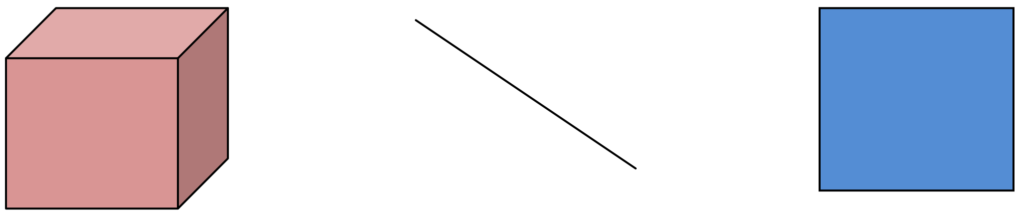
Instructional Mode: Discussion followed by lecture

Date Given: Estimated Time: 30-50mins

Date Submitted to Algebra^3: 3/23/09

An Introduction to Measure Theory Name: ____

Warm-Up: Decide if each object is 1-dimensional, 2-dimensional or 3-dimensional.



Q: Why do we define dimension this way?

We usually consider 1-dimensional measure as length. We can measure this with a ruler.

Q: What happens if we try to measure a square with a ruler?

We usually consider 2-dimensional measure as area.

Q: What happens if we try to find the area of a line segment?

Q: What happens if we try to find the area of a cube?

We usually consider 3-dimensional measure to be volume.

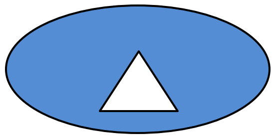
Q: What happens if we try to find the volume of a line segment?

Q: What happens if we try to find the volume of a square?

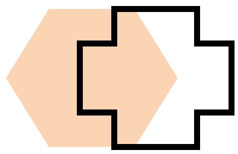
Thus, we define the dimension of an object as the smallest number, n, where the n-dimensional measure is not infinite.

Basic properties of measures: (Write the property illustrated beside each picture.)

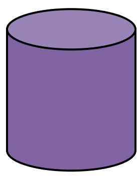
1.



2.



3.

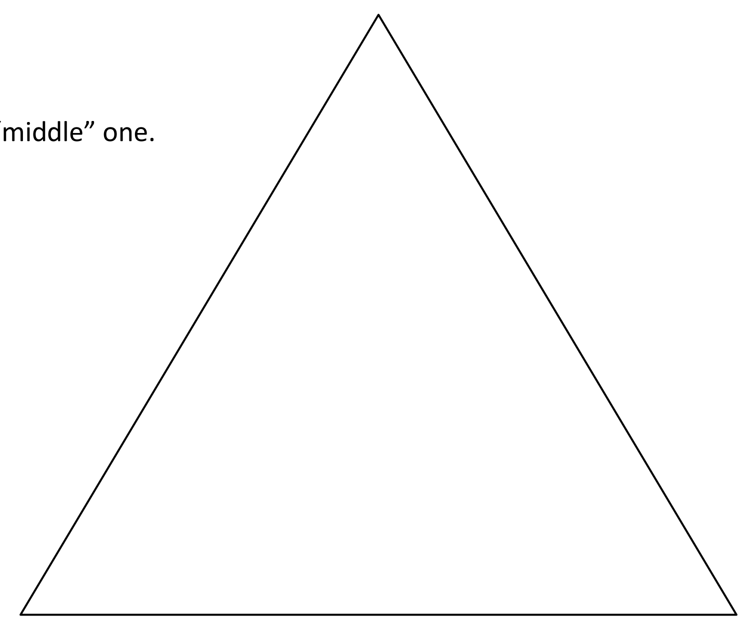


Are there any other ways to measure things?

- Lebesgue Measure: based off of squares (this is the one we are familiar with!)
- Hausdorff Measure: based off of circles
- Harmonic Measure: based off of *functions* (scary!)

Why would anyone care about measuring and dimension beyond what we already know?? (Rhetorical!)

Sierpenski Triangle:



Start with an equilateral triangle.

Split it into four equal triangles, and remove the "middle" one.

(It should look like the Tri-Force!)

Do the same with each triangle you are left with.

Repeat again.

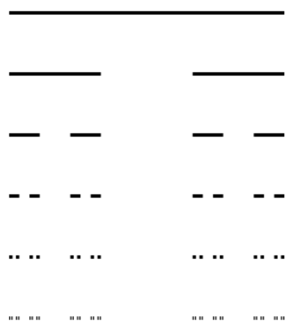
If we continued this forever:

What is the area of the result?

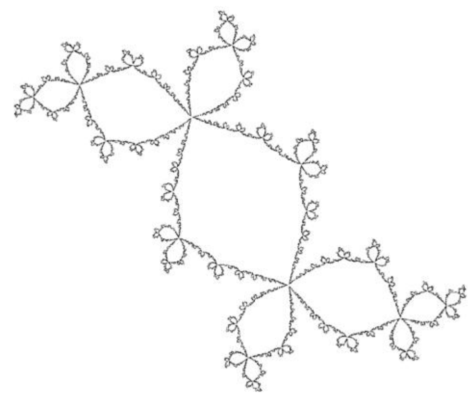
What is the dimension of the result?

Other interesting examples:

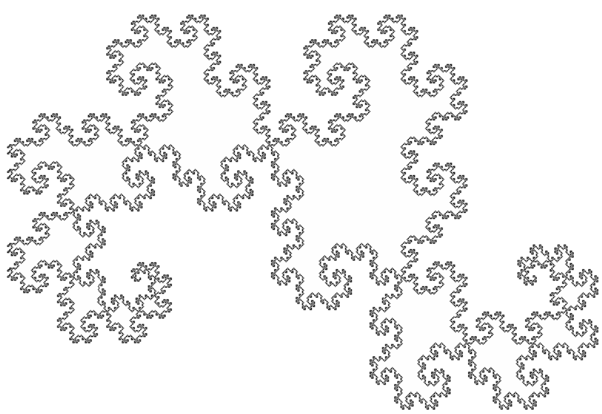
Cantor set (dimension ≈ 0.6309)



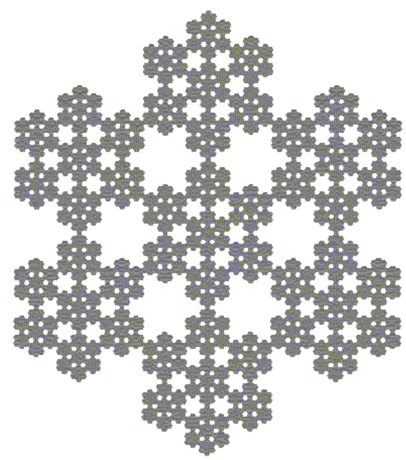
Douday Rabbit (dimension ≈ 1.3934)



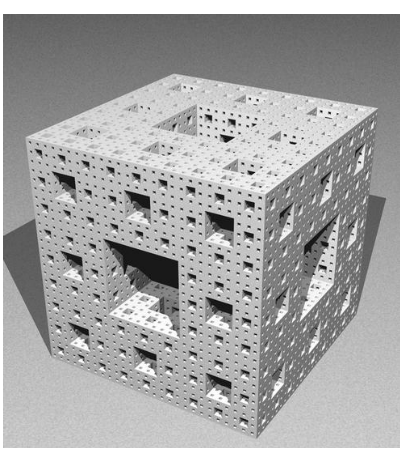
Dragon Curve boundary (dimension ≈ 1.5326)



Hexaflake (dimension ≈ 1.7727)



Menger Sponge (dimension ≈ 2.7268)



Broccoli (dimension ≈ 2.66)