Name: _____ Date: _____

NUMB3RS Activity: Perfect Out-Shuffles
Episode: "Double Down"

In "Double Down," card auto-shufflers in casinos are discussed. In this activity, you will analyze a different type of method for shuffling cards that is not random in nature. This shuffle is used by magicians to mix a deck of cards in a predetermined way, and is commonly called a perfect out-shuffle or a Faro shuffle.

Activity:

Step 1: Take 8 index cards and write the letters A, B, C, D, E, F, G, and H on the cards as shown below (one letter per card).

A	B	C	D	E	F	G	H

Step 2: Arrange the 8 cards in alphabetical order in a pile, with card A at the top and card H at the bottom. Pick up the 8 cards and hold them in your left hand.

Step 3: Take the top four cards in your right hand and leave the bottom four cards in your left hand (i.e. we divided the deck into a half)

Step 4: Place the bottom card of the pile in your left hand on the table. Then place the bottom card of the pile in your right hand on top of the card on the table.

Step 5: Repeat step 4 until all eight of the cards are in a pile on the table.
If you have done the shuffle correctly, the order of the 8 cards from top to bottom is A, E, B, F, C, G, D, and H.
Notice that the cards that were in the top and bottom positions (the outside cards) of the original stack of cards are still in the top and bottom positions of the shuffled stack of cards. This is why this shuffle is called a perfect out-shuffle. (The word perfect is used because the cards are being moved in a predetermined manner instead of randomly.)
If you do this shuffle two more times, the cards will be in the same order as they were at the start! That is, after only three perfect out-shuffles, a deck of 8 cards will return to its original order.

Questions:

1. Look at Table 1 below. The before column displays the order of the 8 cards before being shuffled, and the after column shows the positions of the cards after one shuffle.

Table 1	
Before	**After**
A	A
B	E
C	B
D	F
E	C
F	G
G	D
H	H

Table 2	
x	**y**
1	
2	
3	5
4	
5	
6	
7	
8	

Let x represent the position of a card before the shuffle and the y represent the position afterwards.

For example, note that the card in the third position (the C) ended up in the fifth position. This can be described by saying that if x = 3, then y = 5. Use the information in Table 1 to complete the missing entries in Table 2.

2. Using Table 2, develop a formula that can be used to produce the value of y for all values of x between 1 and 8.

  Hint: Plot the points from Table 2 on the xy-coordinate system.
        Look at the y-values for $1 \le x \le 4$ and $5 \le x \le 8$.
        You will need to use a piecewise function.

3. Let y = f(x), where f is the function that describes the relationship between y and x you found in Question #1. Find each of the following.

x	1	2	3	4	5	6	7	8
f(x) = 1st shuffle								
2nd shuffle								
3rd shuffle								

4. At the start of this activity we saw that the order of a deck of 8 cards will be restored after 3 perfect out-shuffles. Using some index cards, make the following decks.

(a) 2 cards   (b) 4 cards   (c) 16 cards   (d) 32 cards

(i)   Repeat Steps 1- 5 from the activity for each deck and write a piecewise function after one shuffle for each deck as in Question 2. Can you predict the formula for 52 cards in the pile?

(ii)   For each deck, manually perform as many perfect out-shuffles as needed to restore the cards to their original order. Record your answers.

(iii)   Search for a pattern and then predict how many perfect shuffles would be needed for a deck of 64 cards to its original order.

Shuffling and Magicians

• When magicians perform a perfect out-shuffle, they do not perform it using the method described in the activity. Skilled magicians are able to do the entire shuffle while holding the cards in their hands, without using a table. Some magicians are able to do this shuffle with one hand, a feat that leaves people stunned and in awe of their skill. Details about how the deck is held and what the magician actually does can be found at http://web.superb.net/cardtric/sleights/outfaro.htm.
• Another type of shuffle is called a perfect in-shuffle. To perform this shuffle, the first card you put on the table should be the bottom card from the pile of cards in your right hand. Using what you know about perfect out-shuffles, make a prediction about what the result of a perfect in-shuffle would be (think about the positions of the top and bottom cards of the original deck as well as the positions of the top card of each half of the deck). Then check your prediction by using the 8 cards you created for the activity to perform a perfect in-shuffle. What do you notice about the order of the cards? How might a magician use a perfect in-shuffle?
• Magicians most typically use a standard deck of 52 cards. Why might a magician want to perform a perfect out-shuffle? Think about how the magician could take advantage of the fact that the position of the top and bottom cards never change. Also, how might a magician take advantage of the fact that the cards can be secretly restored to their original positions?

Additional Resources
Magic Tricks, Card Shuffling and Dynamic Computer Memories, S. Brent Morris
ISBN 0-88385-527-5, Mathematical Association of America, www.maa.org
The Mathematics of Games, John D. Beasley, Oxford University Press,
ISBN 0-19-286107-7
Ivars Peterson's Math Trek - Magic of Perfect Shuffles - August 1, 1998
http://www.sciencenews.org/sn_arc98/8_1_98/mathland.htm
Shuffle: http://mathworld.wolfram.com/Shuffle.html - includes links to faro shuffle, perfect shuffle, and others.