Probability and Expected Value

Lesson Plan

Cube Fellow: Audrey Brock **Teacher Mentor:** Karyl White

Goal: The goal of this lesson is to introduce students to the methods and ideas behind probability theory.

Theoretical Probability
Students will learn how to find all the possible outcomes of a trial, including repeated outcomes, which are often overlooked. In order to do this, they will make a tree diagram and learn how to make an area model. Once they have determined the possible outcomes, they will compute theoretical probabilities.

Expected Value
Students will make predictions about an experiment based on the theoretical probabilities they found.

Experimental Probability
Finally, students will perform an experiment and calculate the experimental probabilities. They will compare these experimental probabilities with the theoretical probabilities they found earlier, and they will compare the results of the experiment with the expected results

Grade and Course: 7th grade math

KY Standards:
MA-07-4.4.1
Students will apply counting techniques to determine the size of a sample space for a real-world or mathematical situation.

MA-07-4.4.2
Students will:
- **determine theoretical probabilities of simple events**
- **determine probabilities based on the results of an experiment and**
- **make inferences from probability data.**

MA-07-4.4.3
Students will tabulate experimental results from simulations and explain how theoretical and experimental probabilities are related.

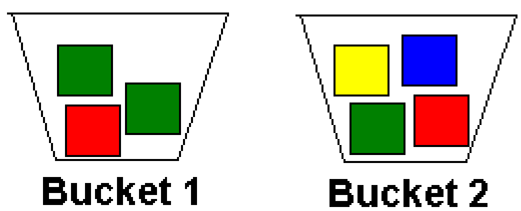
Objectives:

After this lesson students will be able to:

- Differentiate between a theoretical and an experimental probability.
- Draw a tree diagram to find the possible outcomes of a trial.
- Draw an area model to find the possible outcomes of a trial.
- Compute theoretical probabilities.
- Make inferences based on theoretical probabilities.
- Compute experimental probabilities.

Resources/materials needed:

- One quarter
- 1 whiteboard, blackboard, or overhead projector
- 2 buckets or bags per group, labeled "Bucket 1" and "Bucket 2"
- 1 copy of worksheet per group (attached at end of document)
- 2 red, 1 yellow, 1 blue, and 3 green blocks per group, arranged as follows:



Description of Plan:

Prior Knowledge

Before this lesson, students know that the probability of an event is the number of ways that event can happen, divided by the total number of possible outcomes. They are familiar with reducing fractions.

Introduction to Class

When students enter the classroom, the buckets with the cubes in them are sitting in the front of the room to capture the students' curiosity. The lesson begins with a series of class discussion questions intended to get the students thinking. For example:

- What are some carnival games that involve probability?
- What games could we play by flipping this quarter?
- What games could we play with the color cubes in these buckets? (Hold up Bucket 1 and Bucket 2.)

Tell the class that today they are learning about probabilities that can be used in games just like the ones they discussed and elsewhere, and they are going to get the opportunity to perform two experiments, one with a quarter and one with the color cubes.

Introduction to Activity 1 (Coin Flip)
Define THEORETICAL PROBABILITY and EXPERIMENTAL PROBABILITY in writing on the board. If the students usually keep their definitions in the same place, have them write it there. Talk about what the word *theoretical* sounds like (theory). Talk about what word *experimental* sounds like (experiment). Give a couple examples of each type of probability and have the students decide whether they are experimental or theoretical.

Now propose the following situation to the students: *We flip a coin twice. How many possible outcomes are there? What are they?* Write them on the board:

Possible Outcomes: 2 Heads, 2 Tails, A Head and a Tail

Ask the students, *What is the probability of each outcome?* (Students will probably say 1/3.) Explain to the students that 1/3 is not the correct probability in any of the three cases. It seems to make sense, but there is a problem. Sometimes there are hidden outcomes that are repeated, and when dealing with probability, one must be careful to count ALL possible outcomes using a counting technique.

Activity 1 (Coin Flip) Procedure
Show the students how to make a factor tree to list all the possible outcomes of flipping a coin twice on the board:

```
1st flip    2nd flip
 H ------------- H
   ------------- T

 T ------------- H
   ------------- T
```

List the possible outcomes: HH, HT, TH, TT

Now introduce the notation "P(event)" which means "the probability of an event." Find P(HH), P(TT), P(HT), and P(HT) on the board as a class. Discuss how these probabilities were found. Also find P(coins match) and P(coins do not match).

Ask the class, *Are these probabilities theoretical or experimental? Why?*

Announce to the students, "We are going to do our own experiment. I have a quarter, and each person in the class is going to flip it twice. We are going to be divided into two teams like so (girls vs. boys, left half vs. right half, etc.). Each time someone on your team gets a match (two heads or two tails), your team gets a point. If the two flips do not match, you get no point. The team with the most points wins. Since there are ___ students in the classroom, how many people do you expect to get a match? Not a match? How many people do you expect to get HH? TT? HT? TH?" (Write all of the expected values on the board and how the students calculated them. Dwell on this subject for a while and make sure students understand where the expected values come from.)

At this point, go around the room and have each student flip the quarter twice. Have someone at the board record each student's outcome under that student's team name. If pressed for time, all students can flip a penny twice at the same time, but going around the room slowly builds anticipation and gives the students a break in this long lesson.

When the game is done, congratulate the winners, console the losers, and have the students calculate the experimental probabilities of HH, HT, TH, TT, Match, and No Match on their own paper using all of the data, not just their team's data. Make sure they use the notation "P(HH) = ___" rather than just writing down the answer. Have them write a sentence comparing the theoretical probabilities to the experimental probabilities. Were they the same? Were they different? In what way?

Before transitioning to the next activity, discuss why just using an individual team's data would be less reliable than using the entire classroom's data if you were interested in estimating the theoretical probability.

Introduction to Activity 2 (Drawing Blocks)
Tell the class that the next activity will involve the buckets of color cubes that the class talked about earlier. Pose the following questions:

- If you close your eyes and draw a cube from Bucket 1, what is the probability of drawing the red cube? A green cube?
- Now imagine that you close your eyes and draw one cube from Bucket 1 and one cube from Bucket 2. What is the probability of drawing a green and a yellow? Why do you think that?

The concept of an area model is introduced. A square is drawn on the board or overhead, labeled "Bucket 1," and cut into three equal pieces. One third is labeled "R," and the other two are labeled "G." This area model represents Bucket 1. Since there is a 1/3 chance of drawing a red from Bucket 1, "R" gets 1/3 of the area model. Since there is a 2/3 chance of drawing a green from Bucket 1, "G" gets 2/3 of the area model. It should look like this:

Bucket 1
R
G
G

Activity 2 (Drawing Blocks)
The class is split up into pairs. Each pair gets a Bucket 1 and a Bucket 2 with the appropriate color blocks in them. Each pair also gets a copy of the worksheet which is attached at the end of this lesson plan. Guided by the teacher, the students complete the first page of the worksheet at the same rate, discussing what the directions mean along the way. This is intended to teach them how to make an area model, but also how to decode instructions.
Next the class figures out the first probability on the second page of the worksheet together, but then they are allowed enough time to determine the remaining probabilities in pairs. While students are working, walk around the classroom.

The final question on the worksheet relies on a class experiment. Have each student in the class close their eyes, draw a cube from a bucket, and hold it up in the air. (Since there are two students and two buckets in each group, each group should have one cube drawn from Bucket 1 and one cube from Bucket 2.) Record the results for each group, such as "GB" or "RY," etc., on the board. Now the students can finish the last problem on the worksheet.

Closing
Ask the students what they learned about probability. Did they find any of the results surprising? Where could they use what they learned today outside the classroom? Do they know any more techniques for finding all the possible outcomes of a trial? What was the difference between experimental and theoretical probability? Are they usually the same? Are they usually close?

Brain Teaser
If time remains in class, pose the following question: *How could we make a model similar to this area model if we had three buckets to choose from rather than just two?* (More than likely no one will mention making a three-dimensional, cube-like model, but after hearing and discussing their ideas, suggest a three-dimensional model. Describe how each layer of the model would represent a different color of block from the third bucket. It may help to stack cubes directly on top of a student's area model to make your point. Ask if they would still call it an "area" model or something else.)

Lesson Source: Adapted from Investigation 2 of "What Do You Expect?" book from Connected Math series

Instructional Mode: Class discussion, brief lecture to give definitions, class activity, another class discussion, teacher-guided partner activity, class experiment, final class discussion

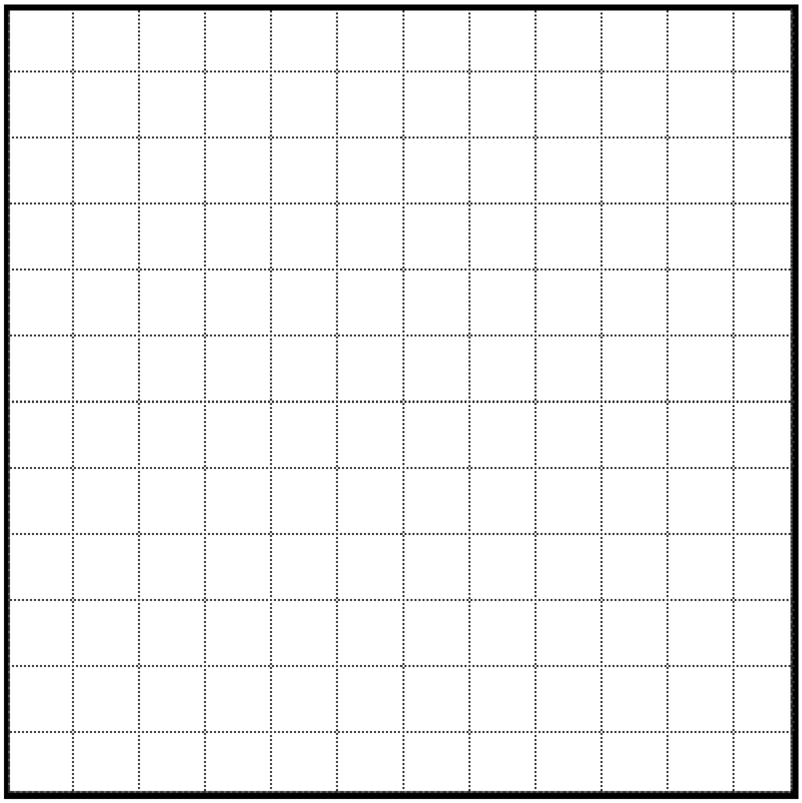
Date Given: 3/23/2007 **Estimated Time**: 1 hour, 20 minutes

Date Submitted to Algebra^3: 4/15/2007

**** Worksheet and Answer Key are on following 4 pages ****

Name: ___________________________

Using an Area Model to Find Probabilities



In Bucket 1 are two green cubes and one red cube. In Bucket 2, there is a red, a blue, a green, and a yellow cube. We are curious about the possible outcomes of blindly drawing one cube from each bucket.

1) The first bucket has three cubes which are equally likely to be chosen.

a) Divide the square above into three horizontal sections with equal areas.
b) Label the sections R, G, and G to represent the red and green cubes.
c) Label the left side of the square **Bucket 1**.

2) The second bucket has four cubes which are equally likely to be chosen.

a) Divide the square above into four vertical sections with equal areas.
b) Label the sections R, B, G, and Y to represent the four color cubes.
c) Label the top of the square **Bucket 2**.

3) Each rectangle formed within the square represents one of these outcomes: RR, RB, RG, RY, GR, GB, GG, or GY, which stand for Red/Red, Red/Blue, Red/Green, etc.

a) Label each rectangle with the matching outcome.

4) Use the area model you drew on the first page to find the following probabilities:

P(RR) =

P(RB) =

P(RG) =

P(RY) =

P(GR) =

P(GB) =

P(GG) =

P(GY) =

P(YY) =

5) What is the probability of drawing a red and a green cube in any order?

6) What is the probability of drawing a red cube from either bucket?

7) When it is time, every group in the class is going to close their eyes and draw one cube from Bucket 1 and one cube from Bucket 2. We will count the number of groups that draw a red cube from either bucket.

a) Based on the number of groups in the class, how many groups do you expect to draw a red cube?

b) Write a sentence comparing the experimental probability of drawing a red cube that we find as a class to the theoretical probability of drawing a red cube that you found in Problem 6.

Name: ANSWER KEY

Using an Area Model to Find Probabilities

Bucket 2

Bucket 1		R	B	G	Y
	R	RR	RB	RG	RY
	G	GR	GB	GG	GY
	G	GR	GB	GG	GY

In Bucket 1 are two green cubes and one red cube. In Bucket 2, there is a red, a blue, a green, and a yellow cube. We are curious about the possible outcomes of blindly drawing one cube from each bucket.

1) The first bucket has three cubes which are equally likely to be chosen.

a) Divide the square above into three horizontal sections with equal areas.
b) Label the sections R, G, and G to represent the red and green cubes.
c) Label the left side of the square **Bucket 1**.

2) The second bucket has four cubes which are equally likely to be chosen.

a) Divide the square above into four vertical sections with equal areas.
b) Label the sections R, B, G, and Y to represent the four color cubes.
c) Label the top of the square **Bucket 2**.

3) Each rectangle formed within the square represents one of these outcomes: RR, RB, RG, RY, GR, GB, GG, or GY, which stand for Red/Red, Red/Blue, Red/Green, etc.

a) Label each rectangle with the matching outcome.

4) Use the area model you drew on the first page to find the following probabilities:

P(RR) = **1/12**

P(RB) = **1/12**

P(RG) = **1/12**

P(RY) = **1/12**

P(GR) = **2/12 = 1/6**

P(GB) = **2/12 = 1/6**

P(GG) = **2/12 = 1/6**

P(GY) = **2/12 = 1/6**

P(YY) = **0**

5) What is the probability of drawing a red and a green cube in any order?

P(RG or GR) = 3/12 = 1/4

6) What is the probability of drawing a red cube from either bucket?

P(RR, RB, RG, RY, or RG) = 6/12 = 1/2

7) When it is time, every group in the class is going to close their eyes and draw one cube from Bucket 1 and one cube from Bucket 2. We will count the number of groups that draw a red cube from either bucket.

a) Based on the number of groups in the class, how many groups do you expect to draw a red cube?

Example: For 10 groups, 1/2 of 10 = 5 groups

b) Write a sentence comparing the experimental probability of drawing a red cube that we find as a class to the theoretical probability of drawing a red cube that you found in Problem 6.

Example: Our experimental probability of drawing a red was 6/10, which is greater than the theoretical probability of 5/10, or 1/2.