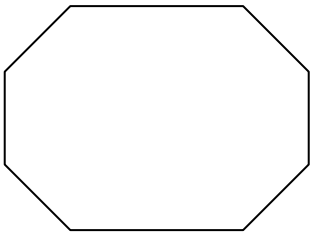
Platonic Solids

A Brief Introduction

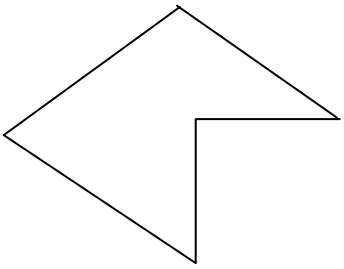
A **polygon** is a two-dimensional shape bounded by straight line segments.

A polygon is said to be **regular** if the edges are of equal length and meet at equal angles.

A polygon is **convex** if the line connecting any two vertices remains inside or on the boundary of the polygon.



Convex



Not Convex

Question 1: Give an example of convex regular polygon.

Question 2: Given any number *n* can you construct a regular polygon with *n* sides?

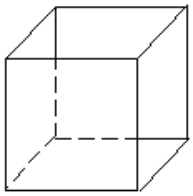
A **Platonic Solid** has the property that each face is an identical convex regular polygon, and that the same number of polygons meets at each corner.

The Platonic solids feature prominently in the philosophy of Plato for whom they are named. The five solids were certainly known to the ancient Greeks and there is evidence that these figures were known long before then.

There are only five Platonic Solids (can you explain why there are only five?)

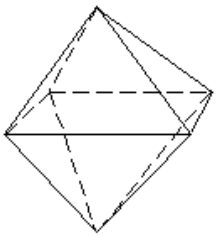
Tetrahedron

Tetrahedron:
Four equilateral triangles, three meeting at each corner.



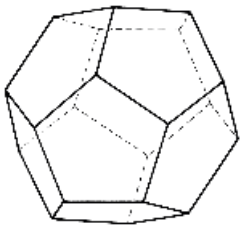
Cube

Cube:
Six squares, three meeting at each corner.



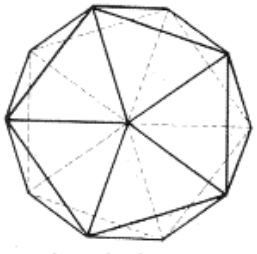
Octahedron

Octahedron:
Eight equilateral triangles, four meeting at each corner.



Dodecahedron

Dodecahedron:
Twelve regular pentagons, three meeting at each corner.



Icosahedron

Icosahedron:
Twenty equilateral triangles, five meeting at each corner.

Activity 1:

Record the following information in the table below:

Platonic Solid	**Vertices**	**Edges**	**Faces**
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

Activity 2:

For each solid compute the following:

Platonic Solid	**Vertices – Edges + Faces**
Tetrahedron	
Cube	
Octahedron	
Dodecahedron	
Icosahedron	

What is your observation? _____

The formula
Vertices – Edges + Faces = "your number from Activity 2"
Is known as *Euler's Formula for Polyhedra.*

Question 3: Do you see any relation between the number of vertices, edges and faces of the platonic solids?

A **polyhedron** (plural: **polyhedra** or **polyhedrons**) is a 3-dimensional geometric shape having flat faces that meet along straight edges.

In geometry, **polyhedra** are associated into pairs called ***duals***, where the vertices of one correspond to the faces of the other.

Platonic Solids are arranged into dual pairs with the exception of one, which is called self-dual.

Question 4: Can you name the dual pairs and the self-dual solid?

Question 5: Suppose you have a cube with side = **x** and you would like to build its dual that fits inside of a cube. What will be the measure of a side of a dual in terms of **x**? Name the dual.

Question 6: Let s = side length of a regular polygon or edge length of a Platonic solid. Find the *surface area* of *each* Platonic Solid.

A **dihedral angle** is the angle of intersection of two planes. In the context of polyhedra, a dihedral angle is the angle of intersection of two adjacent faces. For each of the Platonic solids, there is only one dihedral angle, because all pairs of adjacent faces intersect at the same angle.

The table below gives dihedral angles for each of the solids:

Platonic Solid	**Dihedral angle (θ)**
Tetrahedron	$\cos^{-1}(1/3)$
Cube	$\pi/2$
Octahedron	$\cos^{-1}(-1/3)$
Dodecahedron	$\cos^{-1}(-\sqrt{5}/5)$
Icosahedron	$\cos^{-1}(-\sqrt{5}/3)$

Question 7: Let *s* = side length of a regular polygon or edge length of a Platonic solid. Find the length of a side for the dual of *tetrahedron* and *octahedron*. (Your knowledge of right triangles, the law of sines or the law of cosines will be helpful)

Upon successful answers you can start next activity that is building solids with zometools. Have Fun.