The Tipping Point!

Lesson Plan

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Goal: The primary goal is to illustrate the connections between geometry and real-world problems. Secondary goals are to reinforce understanding of the angle relationships present in parallel lines with tranversals. Also, students will be introduced to the common scientific practice of gathering data through observations and then conducting a mathematical analysis of the data in order to answer meaningful questions. Students will be introduced to a two-dimensional Hele-Shaw cell and learn about the angle of repose of an unconsolidated sediment.

Grade and Course: tenth grade geometry

KY Standards: Geometry: MA-HS-3.1.3 Students will analyze and apply angle relationships (e.g., linear pairs, vertical, complementary, supplementary, corresponding and alternate interior angles) in real-world and mathematical problems.

Objectives: The student will be able to:

1. realize the value of basic geometry for solving meaningful problems
2. understand what is meant by the angle of repose of an unconsolidated sediment
3. carefully gather data from a two-dimensional Hele-Shaw cell by employing a protractor

4. understand that grain-size affects the angle of repose of an unconsolidated sediment

5. find the measure of angles in an unfamiliar setting using their knowledge of parallel lines and transversals.

Resources/Materials Needed: The class will work on this lesson in groups. Each group will need the following:

1. Hele-Shaw cell (instructions for constructing the Hele-Shaw cell may be found at the web site listed under "Lesson Source").

2. protractor.

3. half a cup of sand

4. half a cup of brown rice.

5. a 3×5 index card will make the angle measurement easier to read.

6. Each student will need a copy of the worksheet attached to this document.

Description of Plan: A brief (less than five minutes) lecture introduces key concepts about landslides. Slides are used with dramatic imagery and emphases is put on the fact that eastern Kentucky is an area of high risk from landslides. Roadcuts are a particular danger as they tend to oversteepen hillsides. It is hoped that the lecture will capture attention as well as conveying facts. Next, the students are broken into groups and use a Hele-Shaw cell (see Fig. 1) to measure the angle of repose of dry sand and rice. Work should be delegated within each group so that one member holds

the cell, one pours, one measures, etc. It is important to stress that the cell must be held steady and perpendicular to their desktop while the material is poured (see Fig. 2). The students are asked to carefully measure the angle the pile of material makes with the horizontal (Fig. 3). If time permits, it would be ideal to measure three or more materials with different grain sizes. The students should observe that as grain size increases, so does the angle of repose.

Once all the groups have made their measurements, the data is recorded on the classroom board. The students are asked to use the data to find the average angle of repose for each material. There is an opportunity at this point to explain why multiple observations are better than one.

Finally, the students, still working in groups, complete a worksheet that shows a diagram of a mountain road and hillside in cross-section. They must use their knowledge of geometric relationships along with their observations of the angle of repose of various materials to determine if the road is at risk from landslides. The lesson concludes by discussing the worksheet with the class and comparing results.

Lesson Source: Instructions for constructing the Hele-Shaw cell were found at the following web site:

http://www.seed.slb.com/en/scictr/lab/heleshaw/index.htm

Instructional Mode: The lesson begins with a brief lecture to introduce key concepts followed by a hands-on data-gathering activity. The lesson concludes with group work on applied geometry problems.

Date Given: October 11, 2006.

Estimated Time: It is possible to complete the lesson in a single class period (around 45 minutes). If more time is available, the lesson can eas-

ily be extended by measuring additional materials and by increasing the complexity of the worksheet problems.

Date Submitted to Algebra[3]: October 16, 2006

Figure 1: a Hele-Shaw cell

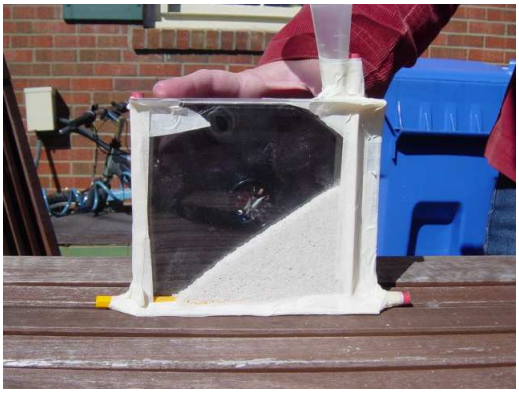


Figure 2: Hele-Shaw cell with an unconsolidated material (sand)

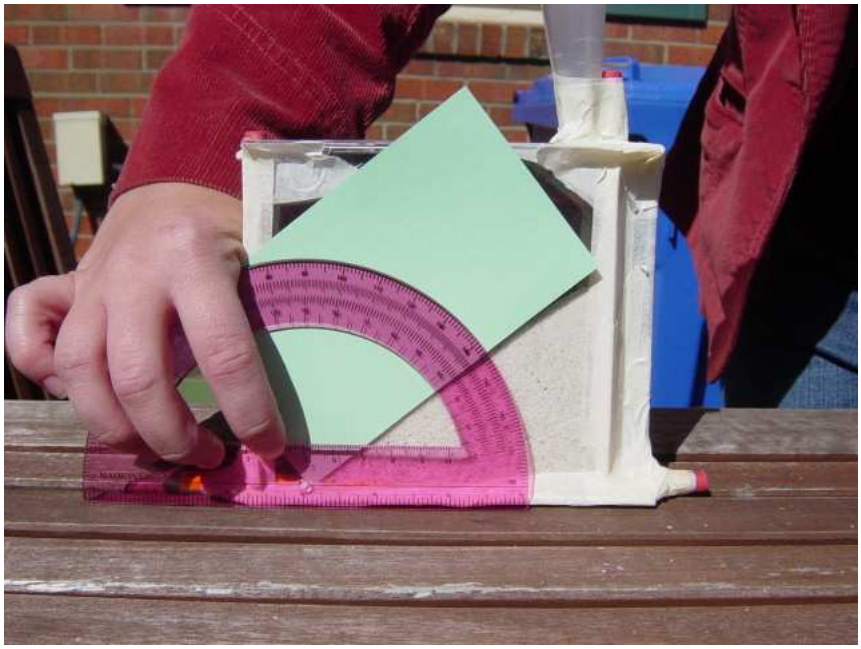


Figure 3: using a protractor to measure the angle of repose

The Tipping Point!

In this lesson we will:

1. Learn about landslides and why they are important.
2. Learn about the *angle of repose* and how it relates to landslides.
3. Use a Hele-Shaw cell to measure the angle of repose of two different materials.
4. Use our geometry skills to decide whether or not a mountain road is safe from landslides!

Figure 4: from http://geochange.er.usgs.gov/sw/impacts/geology/landslides/

Landslides cause between 1 and 2 billion dollars in damage each year in the United States. They are a particularly big problem in the eastern portion of the country.

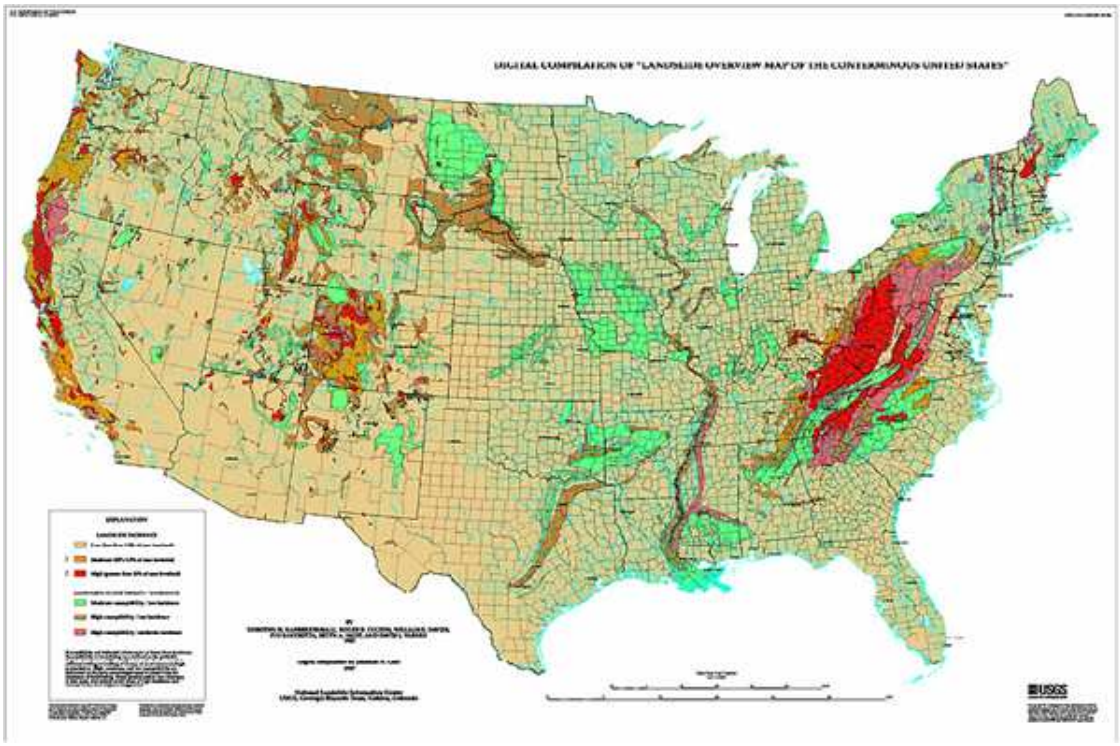


Figure 5: from http://www.usgs.gov/

Roads in mountainous terrain are particularly at risk. Road cuts will often over-steepen slopes, making them dangerous!

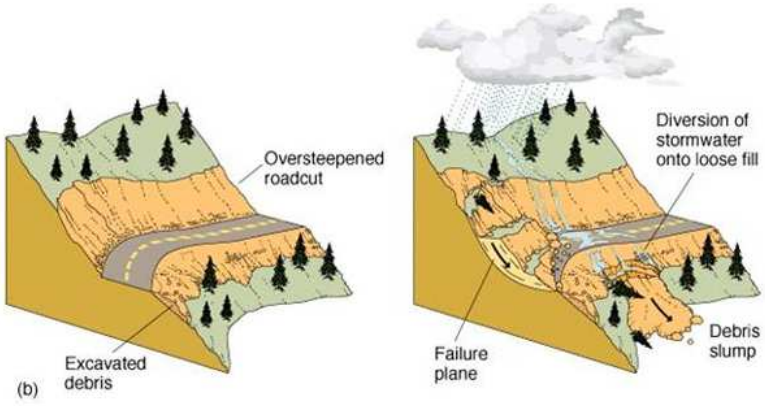


Figure 6: road cut hazards

Figure 7: from http://landslides.usgs.gov/

The Question: How steep is too steep? If we want to make a road cut, how steep can the slop be and still be safe?

The angle of repose is the largest angle at which loose material like sand is stable. The angle of repose is also called the **tipping point**.

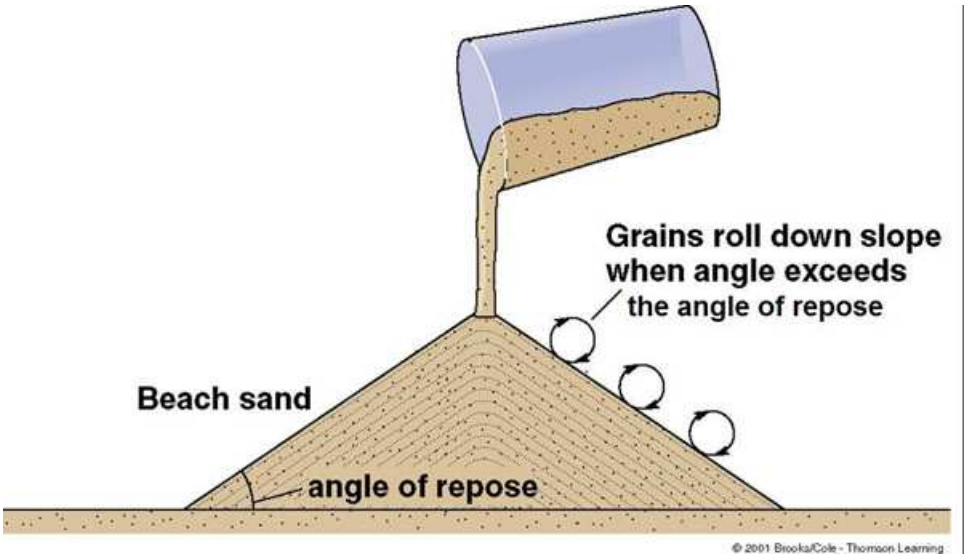


Figure 8: illustrating the angle of repose

The Tipping Point!

Task 1:

Break into groups and use a Hele-Shaw cell and protractor to measure the angle of repose of each material. Note that we will be simulating gravel with rice. The important point is that the rice is more *coarse* than the sand. That means that the individual grains of rice are larger than the individual grains sand. We will see that the *coarseness* will affect the angle of repose.

1. What is the angle of repose of the sand?

2. What is the angle of repose of the "gravel"?

3. What happens to the angle of repose as the size of the particles gets bigger?

4. Each groups observations will be written on the board. Find the average angle of repose of the sand and of the "gravel" based on all of the observations. Complete the table below.

group	sand	gravel
average		

Table 1: angle of repose observations

Task 2:

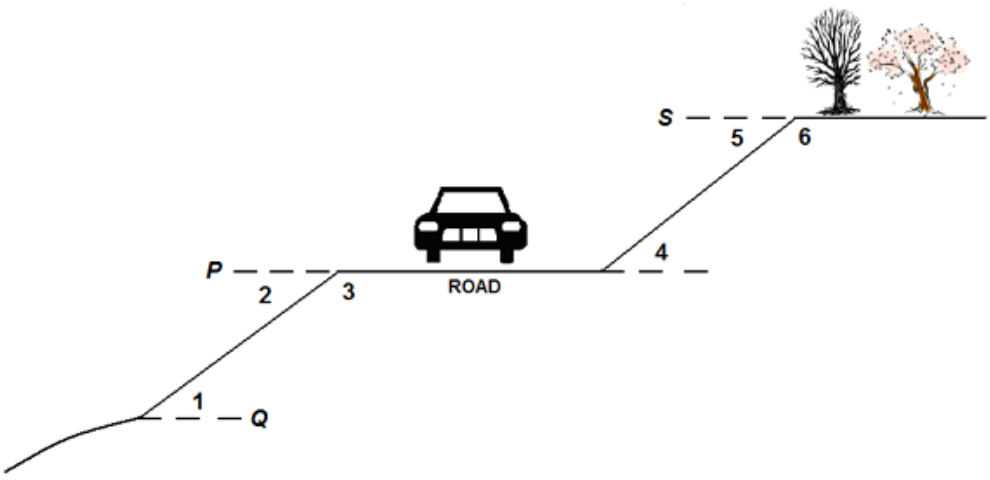
Complete the following worksheet.

The Tipping Point Worksheet

Suppose that we would like to build a road on the side of the hill and are concerned about landslides. We decide that to be safe, the slope must be 5° less than the angle of repose. A diagram of the roadcut is given below:

$P \parallel S$

$Q \parallel P$



1. If the measure of angle **1** is 15°, what are the measures of angles **2** and **3**?

   $m\angle 2 =$ $m\angle 3 =$

2. If the measure of angle **4** is 40°, what are the measures of angles **5** and **6**?

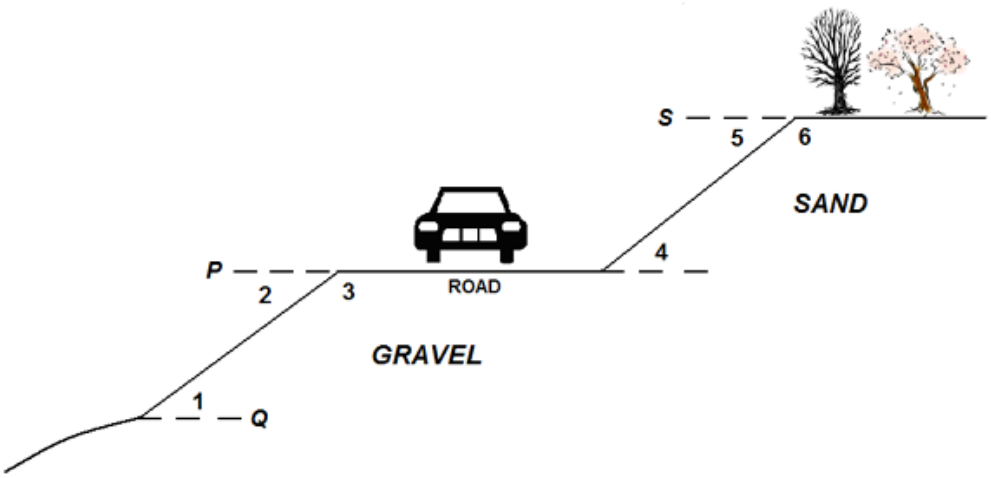
   $m\angle 5 =$ $m\angle 6 =$

3. If the hillside is composed of sand, is the road safe? Why or why not?

Now look at the diagram below. This hill has gravel on the lower slopes and sand on the upper slopes. Answer the following questions.

$P \parallel S$

$Q \parallel P$



4. If the measures of angles **1** and **4** are both 35°, what are the measures of the other angles?

   $m\angle 2 =$ $m\angle 5 =$

   $m\angle 3 =$ $m\angle 6 =$

5. Is the road safe? Why or why not?