Deric's Blotto
Lesson Plan

Cube Fellow: Deric Miller
Teacher Mentor: Dale Adkins
Grade/Class: 8th Grade Math
KY Standards:
MA-08-1.1.3, MA-08-1.4.1, MA-08-4.4.1, MA-08-5.1.5,

Objectives:

Students will gain an introduction to triangular numbers and the Handshake Problem.

Students will engage in an excersise in strategic thought designed to offer them an innate understanding of the nature of ratios, including how changing each of the two numbers of a ratio, both seperately and simultaneously changes the ratio itself.

Resources/Materials needed:

Deric's Blotto spreadsheet file
A computer running either Microsoft Excel or OpenOffice
Two Deric's Blotto gameslips per student
Appropriate prizes for winners

Motivation:

I find many students simultaneously claim to dislike math, but enjoy games of many various types. This lesson seeks to illustrate students at an eighth grade level the ways that games represent interactive mathematical systems, using Colonel Blotto as an example. I hope to convince students both that mathematics underlies most types of games, and that a firm grasp of mathematical principals will make them more competitive at playing games. Colonel Blotto offers an excellent opportunity for students to apply critical thinking and strategic planning skills to the subject of ratios, and an opportunity to explore grade-level appropriate combinatorics, as outlined in the lecture notes, below.

Prior Knowledge:

The lesson assumes that students have had appropriate grade-level exposure to fractions, rates, and ratios.

Lesson Source:

The initial idea of using Blotto games in math education came from a presentation by Andy Niedermaier at the 2009 Joint Maths Meetings in Washington DC. The lesson plan, worksheet, method of staging battles with more than one contender, and excel file presented here are all completely original work.

Mode of Instruction:

Interactive lecture mixed with full participation class game.

Estimated Time:

One class period, eighty minutes in length.

Date Submitted:

4/5/9

Lecture Notes:

Begin by explaining the rules to and story behind the game Colonel Blotto, as follows. Colonel Blotto commands an army consisting of 100 squadrons of soldiers. He has been ordered by his general to use those men to capture ten territorries. The value of these territories ranges from one to ten. In two player game of Colonel Blotto, each player determines, unseen by their opponent, how many armies to deploy to each of the ten territories. Then the deployments are compared, and whichever player deployed more troops to each territory earns territory points equal to the territory's number. In the case of ties, half the points go to each player. Whichever player earns the most territory points wins.

An example game to go over with the class; when determining the winner of each battle, let the students tell you who won and why:

Remember that combatants can not deploy more than 100 squadrons.
John Blotto decided to deploy his 100 squads in the pattern

10	10	10	10	10	10	10	10	10	10
1	2	3	4	5	6	7	8	9	10

Paul Blotto deployed his 100 squads in the pattern

0	0	0	0	0	20	20	20	20	20
1	2	3	4	5	6	7	8	9	10

John won the battles at the first five territories by sending 10 squads to each of them; in each case, those 10 squads had to fight 0 soldiers from Paul, so John wins 1, 2, 3, 4, &5 points, for a total of 15 points. Paul's 20 squads each to territories 6, 7, 8, 9, &10 defeat John's 10 squads each to those territories, giving Paul territories 6, 7, 8, 9, &10 for a total of 40 territory points. Thus, Paul beats John 40 to 15.

Ask the class how many territory points they must aquire to win a round (there are a total of 55 points, thus 27.5 earns them a draw, and any more than that gives them a victory).

At this point, ask the class to generate a strategic deployment of troops, and get them to discuss their strategy, and then move on to pit their strategy in a war against John and Paul, as described below. Replace the example strategy of George with the class's strategy.

When more than two players compete at Colonel Blotto, one method to determine the ranking of players, the method used for this lesson, involves each player's squad deployment battling every other player's deployment in a series of pairwise battles. A player earns one victory point for each battle that they win, and a half point for any battles that they fight to a draw. The player with the most victory points wins the war. Extending the example above, George Blotto enters the war with the deployment

5	5	5	5	30	30	5	5	5	5
1	2	3	4	5	6	7	8	9	10

As already established, Paul's deployment beats John's 40 points to 15. Paul's deployment also beats George's, 34 points to 21. So by beating John and George, Paul earned 2 victory points. John's deployment beats George's, to 44 territory points to 11, thus, the end of war standings come out as follows:

Paul:	2 Victory Points	1st Place
John:	1 Victory Point	2nd Place
George:	0 Victory Points	3rd Place

Draws in these standings can occur.

You have just calculated by hand with the class the victor of three battles, in a larger war, and the final standings of that war. With two players a war involves one battle. Three players requires three battles, and four players requires six battles. The number of battles required for a war involving a given number of Blottos represents an example of the Handshake Problem, and the series of triangular numbers solves the problem. Discuss this with the class; get them to calculate how many battles occur in a war with four players, then five. Observe how much time it would take to determine the winner of a battle involving as many players as you have students as an introduction to the Deric's Blotto spreadsheet.

Get every student to fill out a deployment slip, then either enter those deployments or, preferably, have an aide enter those deployments, with the student's names, into the Deric's Blotto spreadsheet. The spreadsheet comes with the demonstration deployments already in place. Replace them with student names and deployments for the actual class war. Discuss the state of the war as it develops as deployments enter the spreadsheet. Award a delicious candy-type prize to the winner of the war, then lead a discussion on strategy, what worked, what didn't, and why, as a lead in to Round 2.

In the second war of Colonel Blotto, rather than victory going to the combatant who earns the most territory points, victory goes to the combatant who earns the most territory points for every squadron they deploy. For instance, in the first example Paul earned 40 territory points, and deployed 100 squads to earn those points. He therefore earned points at a rate of 40 points / 100 squads, or 0.4 points per squad. In that same battle, John earned 15 points from a deployment of 100 squads, or 0.15 points per squad. In this case, because each of them deployed all 100 of their squads, the result remains unchanged; Paul still wins by the same proportion of points as he would have under the original rules. However, an alternative strategy for victory now comes in to play. The player can decrease the number of squads deployed, the denominator of the fraction and the divisor in the division operation that yields a contestant's final score, and thereby potentially increase that score. An example deployment, by Ringo Blotto, follows.

0	0	0	0	0	26	1	1	1	1
1	2	3	4	5	6	7	8	9	10

Pitted against Paul Blotto's original

0	0	0	0	0	20	20	20	20	20
1	2	3	4	5	6	7	8	9	10

Ringo's deployment wins 0.5 territory points for the tie at territory 1, 1 for territory 2, 1.5 for 3, 2 for 4, 2.5 for 5, & 6 for winning territory 6, for a total score of 13.5, while Paul takes the remaining 41.5 points. However, under the new rules, Ringo's score is 13.5 territory points / 30 squads deployed, or 0.45 points/squad, while Paul's score is 41.5 points / 100 squads deployed, or 0.415 points/squad, so Ringo wins the battle. So now, to win, a player must maximize the ratio of points earned to squads deployed by maximizing territory points while minimizing squads spent. The larger war takes place as a series of pairwise battles exactly as before.

Hand out a new deployment slip to each student. As they turn them in, enter them on the second worksheet of the Deric's Blotto spreadsheet, titled Ratio Blotto. Again, discuss successful and unsuccessful strategies, and award delicious candy to the victor.

If time permits, either stage a third war, using the same rules as the second war, or lead a class discussion on further modifications to the ruleset to produce more interesting games. One such modification to mention is the bomb rule, under which each contestant gets one bomb, which they draw on their game slip above a territory instead of sending troops to that territory. Bombed territories offer no points to either player, so the goal is to use the bomb on the territory that will get them to waste the most soldiers on no territory points.

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10

Deric's Blotto: Write the number of squadrons you wish to deploy above each numbered territory. You only have 100 squads.
Name:

1	2	3	4	5	6	7	8	9	10