Time Dilation and Geometry

Lesson Plan

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Goal: Use basic geometric principles (the Pythagorean Theorem) to discover a special relationship between time and velocity.

Grade and Course: 10-12, Geometry

KY Standards: MA-HS-1.3.1, MA-HS-2.1.3, MA-HS-3.1.5, MA-HS-3.1.7, MA-HS-5.2.1

Objectives: The students should be able to develop a representative right triangle using two basic postulates and a simple thought experiment. The students will then use the Pythagorean Theorem to establish a relationship between specific variables. The main idea here is to get students interested in mathematics by showing them that they can use what they know to develop very surprising and interesting results.

Resources/materials needed: Worksheet and flash animations

Description of Plan: Give brief lecture concerning the material and new ideas, followed by handing out the worksheet. Explain and work through the first part of the problem, and follow up with class discussion, with an emphasis on student participation. Simple animations will be used to help students visualize the thought experiment.

Lesson Source: Original lesson.

Instructional Mode: Brief lecture, followed by class discussion and small group participation on worksheet. The class will also be encouraged to act as a single group by asking questions, and expecting explanations from the class in general.

Date Given: 12/20/2007 **Estimated Time:** One class period

Date Submitted to Algbra^3: 01/14/2008

Geometry Worksheet

Remember the two basic postulates of Special Relativity:

P1) The Galilean principle of relativity holds.

P2) The speed of light in a vacuum is a constant (c), independent of the observer.

Albert Einstein often used what are called "thought experiments" to reason through tricky problems. Suppose that you are observing a rocket ship traveling at a constant velocity in a straight line. Suppose that the rocket's velocity **relative** to you is denoted by *v*. Furthermore, suppose that you observe the rocket for a period of time, *t*.

a) If the following diagram represents the above scenario, then what is the distance traveled by the rocket? Hint: Can you remember the relationship that exists between velocity, time and distance?

Now, we are going to add to our thought experiment by using a "light clock". This clock is an imaginary perfect clock that consists of two parallel mirrors with a photon bouncing between them. Suppose that the mirrors are situated at a distance so that it takes time *t* for a photon to travel between them.

b) Note that the apparent path of the photon as observed by you differs from the path as observed by someone in the rocket. Complete the diagram above, by indicating what path the photon travels from each person's perspective.

c) What does our second postulate tell us about the speed of the photon as observed by you and as observed by the person on the rocket?

d) Now, label the distance on the diagram that the photon travels as observed from the outside of the rocket.

e) But the length of the hypotenuse must be longer than the remaining leg of the triangle, so what does that tell us about the time measured by you, and the time measured by the astronaut? Are they the same? If they are different, denote the time as observed by the astronaut with t_0. Use this information to label the remaining side of our diagram accordingly.

f) Use the Pythagorean Theorem to show the relationship that exists between the three sides.

g) Suppose that the rocket's velocity relative to you is one half the speed of light (so v=c/2), and that you observe the rocket for 1 minute. Then solve the equation in part (e) for the variable t_0.