Polyhedra
Lesson Plan

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Goal: Students will build paper models of various convex polyhedra and count/measure/compute various attributes thereof.

KY Standards: MA-08-2.1.1, MA-08-2.1.4, MA-08-31.3, (MA-08-4.1.1), (MA-08-5.1.2)

Objectives: Students will gain hands-on familiarity with some important and common convex polyhedra including the Platonic solids. They will practice computing surface area of solids and discuss strategies for computing volume. They will also record data in an organized table and look for patterns in the data, perhaps recognizing polyhedron duality or Euler's relation.

Resources and Materials needed: Bell Ringer Puzzle, A Packet consisting of data sheet and polyhedron nets (below), scissors, and tape for each student. Some student time can be saved (at the expense of a great deal of prep time) by cutting out the nets ahead of time. This is not recommended in general, but reusable sets can be made by laminating the facets separately and then assembling the nets.

Description of Plan: First Sheet (Bell Ringer), tape, and scissors should be distributed before or at the start of class. Challenge Students to tape together some or all of the equilateral triangles together edge-to-edge so that exactly two triangles meet at every edge. Allow 10-15 minutes.

The smallest and presumably most likely solution is a tetrahedron (4 triangles), but a triangular bipyramid (6 triangles) is also possible. Watch out for students trying to solve the puzzle with just two triangles, and steer them towards one of the above solutions.

Ask: Can you identify the shape you have made (or you neighbor made)?. Is it a polygon? What is a polyhedron? What is the difference between polygons and polyhedra? Is it possible to follow the rules of the Bell Ringer with all the triangles flat on a table? How or why not? (If students answer no, ask "What if you had infinitely many triangles?")

Polyhedron (pl. polyhedra): From Greek $\pi o\lambda\upsilon\epsilon\delta\rho o\nu$ poly (many) + hedron (seat) Object with many seats (or faces).

Tetrahedron = four faces.

Compute the surface area of the tetrahedron (as a class) using the edge length as a unit. (All of the included polyhedra have edges that are all the same length.) Fill in all the blanks on the data sheet for the tetrahedron. Suggest that students make a special note of the area of an equilateral triangle, as this is useful for other solids.

Distribute the data sheet and prism & bipyramid nets. Have students build the bipyramid. If they already have built this polyhedron, have them build a tetrahedron instead by removing two of of the triangles in the bipyramid net to leave a large triangle subdivided into four smaller ones. As a class, fill in the second row on the data sheet.

Continue as time allows. Volume can be computed "easily" for the tetrahedron, bipyramid, and octahedron with the pyramid formula, and for the prism and cube with the prism formula. The major difficulty here is determining the height of a pyramid. Computing the volume for the icosahedron and dodecahedron would require substantial time and direction from the instructor. Computing the surface area for the dodecahedron is difficult as well, but could be done empirically. (Divide the pentagon into triangles and measure the heights and bases.) One could use the dodecahedron alone for an entire lesson and can serve as either an introduction to, or a tie-in to the golden ratio.

Reserve some time at the end of the class for discussion. Ask students what observations they made, and what patterns they see in the data. Duality (Did anyone notice that some of the polyhedra come in pairs?) and Euler's relation (Can you figure out the number of edges without counting them?) can be seen in the data. Other topics include regular polyhedra (platonic solids), symmetry (which polyhedron is the most symmetric?), convexity (the icosahedral net can be folded into a non-convex icosahedron), and even the definition of polyhedron (*The Original Sin in the theory of polyhedra goes back to Euclid, and through Kepler, Poinsot, Cauchy and many others . . . [in that] at each stage . . .... the writers failed to define what are the 'polyhedra'*

. . . -Grünbaum 1994)

When this lesson was classroom tested in 90 minute periods, we were generally able to construct the first five polyhedra (including the bellringer) and compute all of the surface areas and the volume of the first four, leaving about 5 minutes to discuss duality. A number of students were able to work ahead on the building and complete the Dodecahedron and Icosahedron as well. However, one class had not yet studied special right triangles or similar triangles and this led to a far less productive class. If this is the case, I do not recommend trying to compute volume except for the pyramid and cube. It is hard enough to find the area of an equilateral triangle when ones only tool is the Pythagorean Theorem. Instead I suggest skipping the surface area and volume portions and focus on duality, Euler's formula, and possible regularity and symmetry.

Lesson Source: Independently created lesson.
Instructional Mode: Guided Hands-on activity
Date Given: 8 October 2008 **Estimated Time:** minimum 40-45 minutes, but probably not suited to multiple sessions

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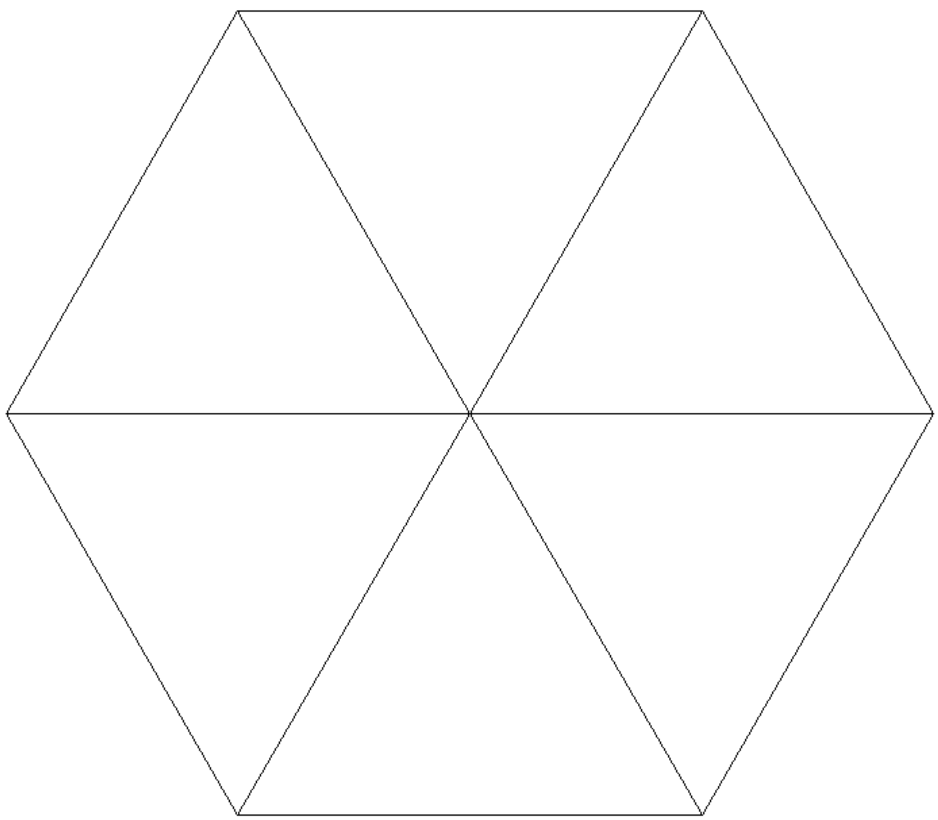
Bell Ringer

Cut out some or all of the six equilateral triangles below. You may leave some attached to each other if you wish. Tape some or all of the triangles together, edge to edge so that your result follows these rules:

- No triangles are bent, torn, or twisted.
- There are more than two triangles involved.
- Every edge of every triangle is taped or attached to exactly one other edge.



Polyhedra

Date: Name:

Use this table to record your observations and calculations. You will not necessarily fill in all the blanks. Record any other observations (patterns, relationships between polyhedra, etc) below.

Name(s)	faces	vertices	edges	vertices per face	faces per vertex	surface area	volume

Observations:

Cut along the outer solid lines.
Fold along the inner dotted lines.
Tape edges together to form
(the boundary of) a solid.