Why Doesn't SSA Work?
Lesson Plan

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Grade/Course: 10th /Honors Geometry
KY Standards:
MA-HS-3.1.6 Students will know the definitions and basic properties of a circle and will use them to prove basic theorems and solve problems.
MA-HS-3.1.12 Students will apply the concepts of congruence and similarity to solve real-world and mathematical problems.

Objectives:

- The student will prove that SSA is not a valid congruence relationship for triangles.
- The student will deduce from the proof what conditions make SSA valid.
- The student will manipulate geometric tools (compass, straight edge) to verify geometric identities.
- The student will exercise their knowledge of segments, radii, reflexive property, rays, angles and circles to discover properties about triangles.

Resources/Materials needed:

- Compass
- Ruler
- Construction Paper
- Markers
- Pipe Cleaners
- Scissors
- SSA Worksheet (attached)

Motivation

Students are often confused about why SSA, AAA are not congruence relations for triangles. While it is easy to give an example for AAA (take an equilateral triangle with sides of length 1, then another with sides of length 2), it is often hard for teachers to come up with a good example for illustrating that SSA does not work.

Mrs. Callahan explained that her students wanted to know why "ASS" was not a valid congruence relation. She asked me to develop an exercise that would show students why. This led to a great learning opportunity, because SSA actually does work sometimes! (But not always.)

Prior Knowledge

Students are assumed to have a solid understanding of rays, segments, angles, circles and the reflexive property. Students should be given this activity after a lesson on triangle congruence by (ASA, SSS, SAS). Students should also understand how to use a compass, or a brief explanation should be given before handing out the worksheet.

Experience with geometric proofs would be preferred, although not necessary. The class I used this activity with had never proved nor seen a geometric proof before. This is an excellent example of how to prove something by contradiction. It is a friendly environment for students to begin proof by contradiction because the steps are outlined completely, and the contradiction at the end is very clear.

Outline of Lesson

I. Depending on the level of maturity of the classroom, this activity can be done individually, in partners or in groups of three. Any larger sized groups might not be beneficial to the students because there are not enough jobs to do.
II. Students should read materials list, and send a materials collector to pick up materials.
III. Students should read through the directions, and follow them one step at a time.
IV. Teacher should circulate, helping students as needed. When I used this activity, step #5 on the first part was the hardest to understand. They wanted to draw two different rays, even though the instructions said one.
V. The most important part is the summary at the end.
VI. As a finisher for the classroom, the teacher should wrap up the activity with an explanation.
    a. Teacher should draw the two triangles from the first part on the board. (Preferably in different colors to illustrate the different triangles, since one is inside the other) Next the teacher should label the SSA, stressing that the middle "S" is the side AB.

b. The teacher should ask the students questions about why the two triangles are not congruent. "Because one is inside the other, and it is obviously smaller", or "Because BD is not congruent to BF" are examples of good answers.
c. The teacher says something like, "But we assumed that SSA was a good congruence relationship, and these two triangles clearly satisfy SSA because we built them that way. What do you think this means?"
d. Students should respond with something like, "Well they can't be congruent because there is no way that BD is congruent to BF, so our assumption must have been wrong."
e. The teacher should now draw the figure from the second part on the board, and explain that this method does create two congruent triangles when it is repeated. The teacher should ask students the difference between the first part and second part.
f. Because the students have the worksheet completed, they will know that they should compare side AB and the other given side in the SSA relation. The students will then realize the big conclusion:

"If the middle 'S' is longer than the first 'S', then SSA does not work. If the middle 'S' is shorter than the first 'S', then SSA does work."

If desired, the teacher can also discuss how the Hypotenuse-Leg (HL) congruence relation is another specific example of this. The middle 'S' is always shorter than the first 'S' because the hypotenuse is the first 'S' and is always longer than the other individual legs.

Note that the wrap up conversation at the end is quite necessary. The students often need guidance on seeing the two congruent triangles in the part one case. They also need more explanation about comparing part one with part two.

Suggestion for follow up:
Create a bell-ringer for the next day's class by drawing several triangles on the board with different leg lengths, some which can be congruent by SSA and some which cannot. Also mix in other congruence relations and ask students to identify which are being used to tell if the triangles are congruent.

Lesson Source
I created this lesson based on a suggestion from Dr. Jubal at The Math Forum.

http://mathforum.org/library/drmath/view/54659.html

Dr. Jubal suggested the activity which I adapted to use for the classroom. I incorporated the idea of using the pipe cleaners because it was a tangible way for students to compare the lengths of the radii. Rather than just tell them that the sides were radii, I wanted them to deduce the reasoning on their own.

I also incorporated the idea of using markers/construction paper/partners to make the activity more appealing to high school students.

Mode of Instruction

This activity should follow a lecture on triangle congruence. This can follow immediately after the lecture, or on the next day of class. This is a very interactive lesson with a wrap up explanation at the end. It is important that the students use self-discovery with these guided instructions and that the teacher monitors the activity closely to make sure it is truly self-discovery and not just copying off of the nearby groups.

Date of Implementation/ Estimated Time

Tuesday, February 17, 2009/ 50 minutes

Date Submitted to Algebra^3

Tuesday, February 17, 2009

attachment: SSA activity

Names: _____

In this activity you and your partner(s) will be constructing a figure to show that SSA is not a valid way to show that two triangles are congruent. You will need construction paper, a compass, a straightedge, a marker/crayon, scissors, and a pipe-cleaner.

Follow the instructions below carefully and answer the questions at the end. You should turn in this paper along with your construction paper sheet. Please raise your hand if you have any questions!

Suppose that SSA **is** a valid way to tell if triangles are congruent. Then if we construct two triangles which have the same SSA, they should be congruent. Let's try it:

Part I

1. On one side of the paper draw two points, label them A and B. You should draw A close to the center and not draw B more than 4 inches away from A.

2. Connect A and B with a straight line using your straightedge. (This side will be the middle "S" in SSA).

3. Choose a point between A and B on the segment AB. Label this point C.

4. Now, take your compass, and placing the spike end on A and the pencil end on C, swing the compass around to make a complete circle.

5. Next draw one ray beginning at point B that touches the circle at two points. You can draw this ray in any direction except back on top of line segment AB.

6. Label one of the points where the ray intersects the circle D and the other point F.

7. Label <ABD with an E. Angle E is the "A" in SSA. (Notice that <ABD is equal to <ABF is equal to angle E).

8. Now, take your pipe cleaner and cut it to fit segment AC. This is the radius of your circle. Fix the pipe cleaner at point A and swing the other end around to touch D. Now connect A and D with a line to make the segment AD.

9. Finally lay the pipe cleaner back on AC, fix one end at A and swing the other end around to touch point F. Now, connect AF with a line.

10. Notice that you have two triangles, triangle ABD and triangle ABF.

11. Notice that segment AD is congruent to segment AF because they are _____ of circle A.

12. Notice that AB is congruent to AB by the _____ property.
13. Notice that angle E is congruent to angle E by the _____property.

14. So, if SSA were a true congruency property, then we would have BF congruent to BD. However this is clearly not true. (If you don't believe this, measure them!) (This is a contradiction to assuming SSA works.)
15. You have just shown that SSA does not work. :)

Follow these instructions to show that *sometimes* SSA does work.

Part II

1. On the other side of the paper draw two points, label them A and B. You should draw A close to the center and not draw B more than 3 inches away from A.

2. Connect A and B with a straight line using your straightedge. (This side will be the middle "S" in SSA).

3. Now extend your line segment AB past point B with a dotted line no more than one inch past B. Label this new ending point C.

4. Now, take your compass, and placing the spike end on A and the pencil end on C, swing the compass around to make a complete circle.

5. Now, draw a ray beginning at point B and extending to the outside of the circle. You can draw this ray in any direction except back on top of line segment AB. Label the point where this ray intersects the circle D.

6. Label <ABD with an E. Angle E is the "A" in SSA.

7. To make the final side of your triangle, take your pipe cleaner and cut it to fit segment AC. This is the radius of your circle.

8. Now, fix the pipe-cleaner at point A and swing AC around until the "C" end touches point D. In this way, you can connect points A and D to make the third side of your triangle.

9. Notice that there is only one triangle to look at, triangle ABD.

10. This shows us that if we drew another identical picture, then our new triangle formed in this way would be congruent to triangle ABD.

11. So, to summarize the two pictures, SSA does **not** work when the middle "S" (in your pictures this "S" is AB) is _____ than the other side.

(Hint: Part I is where SSA did not work)

12. SSA **does** work when the middle "S" (this "S" is AB) is _____ than the other side.

(Hint: Part II is where SSA does work)

(Fill in the above with either "shorter" or "longer")

13. Did you enjoy this activity?

14. Comments: