Topic

Average Rate of Change of \( f(x) \) from \( x=a \) to \( x=b \)

\[ \frac{\Delta f}{\Delta x} = \frac{f(b)-f(a)}{b-a} \]

Also Known As

Average Speed

Average Velocity

Interpretation

Slope of secant line from \((a,f(a))\) to \((b,f(b))\)

Derivative of \( f(x) \) at \( x=c \)

\[ \lim_{h \to 0} \frac{f(c+h)-f(c)}{h} \]

\( \text{Note: to find a formula for the derivative, find} \)

\[ \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \]

\( \text{Instantaneous Rate of Change at } x=c \)

\( f'(c) \)

\( y' \)

\( \frac{dy}{dx} \)

\( \frac{df}{dx} \)

\( \text{Slope of tangent line to } f(x) \text{ at } x=c \)
**Topic**

Limit of $f(x)$ as $x$ approaches $c$

$$\lim_{{x \to c}} f(x)$$

**Interpretation**

- Value of $f(x)$ as $x$ values get closer & closer to $c$.
- If $f(x)$ is a polynomial, then $\lim_{{x \to c}} f(x) = f(c)$ (just plug in $c$).
- $\lim_{{x \to c}} f(x)$ exists only if $\lim_{{x \to c^+}} f(x) = \lim_{{x \to c^-}} f(x)$.
**Topic**

Continuity - 

$f(x)$ is continuous at $x = c$ if

$$\lim_{{x \to c}} f(x) = f(c)$$

**Interpretation**

- The limit as $x$ approaches $c$ is the same as plugging $c$ into the function.
- Geometrically, no holes, jumps or gaps.
- Continuity on an interval means $f(x)$ is continuous at every point in the interval.
- Polynomials are continuous.
- Rational functions are continuous at every point in their domain.
**Topic**

Differentiability - 

$f(x)$ is differentiable at $x = c$ if the derivative exists at $x = c$

(i.e. $\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$ exists)

**Interpretation**

- The tangent line exists and its slope exists.
- Geometrically, no sharp corners or jumps.
- Differentiable on an interval means $f(x)$ is differentiable at every point in the interval.

- If $f(x)$ is diff. at $x = c$, then $f(x)$ is cont. at $x = c$.
- Polynomials & Rational Ftns are differentiable.
**Topic**

Intermediate Value Theorem (IVT)

**Interpretation**

If \( f(x) \) is continuous on the interval \([a, b]\), then for every value \( k \) between \( f(a) \) and \( f(b) \), there exists \( c \) between \( a \) and \( b \) such that \( f(c) = k \).
Derivatives We know so far:

\[
f(x) = ax^2 + bx + c
\]

\[
f'(x) = \lim_{{h \to 0}} \frac{f(x+h) - f(x)}{h}
\]

\[
f'(x) = 2ax + b
\]

\[
f'(x) = m
\]
Topic
Eqtn for the tangent line to $f(x)$ at $x=a$

Idea
For an equation of a line, we need the slope and a point on the line.
Slope: $m = f'(a)$
Point on the line: $(a, f(a))$
Point-slope formula:
$y - y_1 = m(x - x_1)$

This gives an equation for the tangent line: $y - f(a) = f'(a)(x - a)$