

Name:
Date:
Time Begun:
Time Ended:

**MA 515 Linear Programming
and Combinatorial Optimization
Fall 2008, Exam 2**

This exam is a take-home exam. There are 120 possible points. You are allowed two hours to complete it and are not allowed to use your book or notes. You may not use separate blocks of time totalling two hours; you are to take this exam in a single two-hour block of time. You should return this exam to the instructor by the start of class on Friday, Nov 21, 2008.

You are not to discuss any aspect of this exam with anyone until after class on Friday, Nov 21, 2008.

Prove one of the following two theorems, each worth 20 points.

Problem 1a: State and prove Hall's theorem. Be detailed. You may assume that König's theorem is true.

Problem 1b: State the weighted greedy algorithm for weighted matroids and prove that it produces maximum weight independent sets. Be detailed.

Complete five of the following six problems, each worth 20 points.

Problem 2a: Prove or disprove: If the linear program

$$\begin{aligned} \max c^T x \\ Ax \leq b \\ x \geq 0 \end{aligned}$$

is unbounded, then there is a subscript k such that the linear program

$$\begin{aligned} \max x_k \\ Ax \leq b \\ x \geq 0 \end{aligned}$$

is unbounded.

Problem 2b: Prove that the vertex-edge incidence matrix for a graph G , with no loops or multiple edges, is totally unimodular.

Problem 2c: Given a graphic matroid M , prove directly that the following holds:

$X \in \mathcal{C}(M), Y \in \mathcal{C}(M), X \neq Y, e \in X \cap Y \implies \exists Z \subset (X \cup Y) - e$ s.t. $Z \in \mathcal{C}(M)$.

Problem 2d: Prove that the Fano matroid is not graphic.

Problem 2e: Provide an example of an edge-weighted graph with 8 edges and 5 vertices. Use the greedy algorithm on the associated graphic matroid to find a maximum weight spanning tree. Show each step of the algorithm.

Problem 2f: Give a complete proof that the dual of a linear matroid is a linear matroid. You may assume any theorems from linear algebra that you find useful but not any homework problems from this class.