Name:
Date:

MA 515, Linear Programming and Combinatorial Optimization Fall 2008, Final Exam

There are 150 possible points. You are not allowed to use your book or notes.

Problem 1: (30 points) First, describe the main themes of this course and their interactions. Second, choose a theorem that you feel excellently represents these themes and do the following:

(1) State the theorem;
(2) Give an example of the theorem in action; and
(3) Explain why you chose this theorem, specifically addressing how it relates to the main themes you described in the first part.

Problem 2: (40 points) Consider the following five contexts in which we have a "max=min" result.

(1) Linear programming
(2) Greedy algorithm for matroids
(3) Matroid intersection
(4) Bipartite matchings
(5) Matchings in general graphs

Choose *four* of these and make a precise statement of the "max=min" result in each case. Sketch the proof for *one* of these, indicating clearly which one you have chosen.

Complete four of the following five problems, each worth 20 points.

Problem 3a: Consider the following matrix:

$$A := \begin{bmatrix} -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Let M be the linear matroid determined by A. Provide a description of the matroid polytope for M as an $\mathcal{H}$-polytope, and use linear programming to find a maximal independent set in M.

Problem 3b: Construct a graph G on 6 vertices with 9 edges. Assign to each of these edges a distinct positive weight. Choose a specific vertex, label it v, and use Dijkstra's algorithm to find all minimum-weight $v-w$-dipaths in G.

Problem 3c: Let G be a planar graph. Take any planar embedding of G and form the planar dual G^*. Prove that the graphic matroid of G^* is the dual of the graphic matroid of G.

Problem 3d: Let G be a graph with no loops and no multiple edges. Prove that a matching S of G is of maximum cardinality if and only if G has no augmenting path with respect to S.

Problem 3e: Given two matroids M_1 and M_2 on a common ground set E and an independent set $S \in \mathcal{I}(M_1) \cap \mathcal{I}(M_2)$, define the bipartite augmentation digraph for S and explain its role in the Cardinality Matroid-Intersection Algorithm.