Linear and Combinatorial Optimization[1]
MA/STA 515, Section 001, Fall 2008

1. General Information

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2. Texts

2.1. **Required Texts:** *A First Course in Combinatorial Optimization*, by Jon Lee. Cambridge University Press, 2004.

Linear Programming Notes, by Carl Lee. Available from the course website.

3. Course Description and Goals

> The best way to teach real mathematics, I believe, is to start deeper down, with the elementary ideas of number and space. . . . in fact, arithmetic, algebra, and geometry can never be outgrown. . . by maintaining ties between these disciplines, it is possible to present a more unified view of mathematics, yet at the same time to include more spice and variety.
>
> *Numbers and Geometry*
> John Stillwell

Finding optimal solutions to mathematical models is a classical problem; even first-year calculus students solve problems involving local and global maximization and minimization! Linear programming is the study of finding maximal and minimal solutions to systems constrained by linear equalities and inequalities. Problems of this type arise in many disciplines, including mathematics, statistics, chemistry, computer science, economics, and others. Combinatorial optimization problems arise when we seek to find maximal and minimal solutions to some model over a finite object, typically one with combinatorial structure. The interplay of linear programming and combinatorial optimization leads to beautiful mathematics and rich applications.

We will discuss the following sections from the course texts:

- C. Lee, *Linear Programming Notes*
- J. Lee, Chapter 0, Linear Programming
- J. Lee, Chapter 1, Matroids
- J. Lee, Chapter 2, Minimum-Weight Dipaths
- J. Lee, Chapter 3, Matroid Intersection
- J. Lee, Chapter 4, Matching
- J. Lee, Chapter 5, Flows and Cuts
- Additional Topics, as time permits

My goal as an instructor in this course is to nurture your innate desire to discover and understand mathematical truths. My goal for you, the student, is to achieve profound understanding of linear and combinatorial optimization[2]. By this, I mean that you will:

[1] I reserve the right to change or amend this syllabus at any time for any reason.

[2] The following description is adapted from the work of Liping Ma, as described in her book *Knowing and Teaching Elementary Mathematics*; see page 122 in particular.

(1) make connections among relevant mathematical concepts and procedures, from simple and superficial connections between individual pieces of knowledge to complicated and underlying connections among linear algebra, combinatorics, polyhedral geometry, and related areas;
(2) approach problems from multiple perspectives, providing explanations of the varying perspectives and comparing their advantages and disadvantages;
(3) be aware of simple but powerful basic mathematical concepts and principles such as the roles and purposes of linearity, maximality, etc.; and
(4) relate ideas and techniques from linear and combinatorial optimization to previous mathematical study.

In short, you will know how to solve problems in linear and combinatorial optimization, know why your solutions work, and understand how they are related to the mathematical world and beyond.

4. **Course Assessment**

- You must be present and engaged in class discussion each day. If you need to miss class for some reason, please notify me ahead of time.
- There will be regular homework assignments. These will be graded and returned to you. *WARNING:* No late work will be accepted.
- While searching the library or internet for solutions is not allowed, you may work on homework problems with your classmates (mathematics is a social endeavor). For each homework problem, list the people you worked with. You must write up your own answers to all the questions; do not let cooperation degenerate into one person solving the problem and other people copying their answers. The act of copying a written answer from another student and submitting it as your own will be considered cheating and will be dealt with according to the procedures referenced in Section 6.
- There will be two in-class exams and a cumulative final.

5. **Course Grades**

Graduate Students: Your total grade will be determined by your homework and exams. The grading scale will be no stricter than the usual A>89.9, B>79.9, C>69.9, D>59.9, E otherwise, weighted as follows:

- Homework: 30%
- In-Class Exams: 20% each
- Final Exam: 30%

Undergraduate Students: Your total grade will be determined by your homework and exams. The grading scale will be no stricter than A>84.9, B>74.9, C>64.9, D>54.9, E otherwise, weighted as follows:

- Homework: 30%
- In-Class Exams: 20% each
- Final Exam: 30%

6. **Academic Integrity and Classroom Demeanor**

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

http://www.uky.edu/USC/New/SenateRulesMain.htm

Turn off all cell phones, pagers, etc. prior to entering the classroom. ***You are not to use your cell phones, pagers, or other electronic devices during class.*** An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

7. **Classroom and Learning Accommodations**

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.

8. **Tentative Schedule**

In the following, "Notes" refers to the notes by Carl Lee and "Opt" refers to the optimization book by Jon Lee. Thus, a reference to Notes, 7.3 means section 7.3 in Carl Lee's Notes.

IMPORTANT COMMENT: The Notes by Carl Lee are a more detailed presentation of the material in Chapter 0 of Jon Lee's book. You are expected to look through Chapter 0 and familiarize yourself with both presentations. This will help you when we switch to using Jon Lee's book exclusively.

- Wed, Aug 27: Course Introduction and Notes, 8.1 – Matrices
- Fri, Aug 29: Notes, 8.2 and 8.3 – Matrix Algebra, Graphs and Digraphs
- Mon, Sept 1: LABOR DAY HOLIDAY, No class
- Wed, Sept 3: Notes, 3.1 – V-polytopes
- Fri, Sept 5: Notes, 3.2 – H-polytopes
- Mon, Sept 8: Notes, 3.3 – H-polytopes are V-Polytopes
- Wed, Sept 10: Notes, 3.4 – V-Polytopes are H-Polytopes
- Fri, Sept 12: Notes, 4.1 and 4.2 – Systems of Equations and Fourier-Motzkin Elimination
- Mon, Sept 15: Notes, 4.3 and 4.4 – Feasibility and Infeasibility examples
- Wed, Sept 17: Notes, 4.5 and 4.6 – Fourier-Motzkin in General
- Fri, Sept 19: Notes, 5, 6.1, 6.2, and 6.3 – Vertices, Introduction to Linear Programming
- Mon, Sept 22: Notes, 7.1 and 7.2 – Dual Linear Programs
- Wed, Sept 24: Notes, 7.3 and 7.4 – Duality Theorems
- Fri, Sept 26: Notes, 7.5, 7.6 and 7.7 – Complementary Slackness, Duals of General LPs, and Geometric Motivation
- Mon, Sept 29: Notes, 8.4 and 8.5 – Systems of Equations and Solving Linear Programs
- Wed, Oct 1: Notes, 8.6 – The Revised Simplex Method
- Fri, Oct 3: Opt, 0.8 – Totally Unimodular Matrices
- Mon, Oct 6: Opt, 0.8 – Graphs, Digraphs, König's Theorem and Hall's Theorem
- Wed, Oct 8: Review
- Fri, Oct 10: Exam 1
- Mon, Oct 13: Opt, 1.1 – Matroids
- Wed, Oct 15: Opt 1.2 – Circuit Properties
- Fri, Oct 17: Opt 1.3 – Representations
- Mon, Oct 20: Opt 1.4 – The Greedy Algorithm
- Wed, Oct 22: Opt 1.5 – Rank Properties
- Fri, Oct 24: Opt 1.6 – Duality
- Mon, Oct 27: Opt 1.7 – The Matroid Polytope
- Wed, Oct 29: Opt 2.1 – No Negative-Weight Cycles
- Fri, Oct 31: Opt 2.2 – All-Pairs Minimum-Weight Dipaths
- Mon, Nov 3: Opt 2.3 – Nonnegative Weights
- TUESDAY, NOVEMBER 4: ELECTION DAY – No classes
- Wed, Nov 5: Opt 3.1 – Applications of Matroid Intersection
- Fri, Nov 7: Opt 3.2 – Efficient Cardinality Matroid-Intersection Algorithm

- Mon, Nov 10: Opt 3.2 and 3.4 – The Matroid Intersection Polytope
- Wed, Nov 12: Review
- Fri, Nov 14: Exam 2

- Mon, Nov 17: Opt 4.1 – Augmenting Paths and Matchings
- Wed, Nov 19: Opt 4.2 – The Matching Polytope
- Fri, Nov 21: Opt 4.3 – Duality and a Maximum-Cardinality Matching Algorithm

- Mon, Nov 24: Opt 5.1 – Source-Sink Flows and Cuts
- Wed, Nov 26: THANKSGIVING HOLIDAY, No class
- Fri, Nov 28: THANKSGIVING HOLIDAY, No class

- Mon, Dec 1: Opt 5.2 – An Efficient Maximum-Flow Algorithm and Consequences
- Wed, Dec 3: A Panorama of Combinatorics
- Fri, Dec 5: Review

- Mon, Dec 8: Putting it all in perspective – What have we done?
- Wed, Dec 10: Another Panorama of Combinatorics – Where do we go from here?
- Wed, Dec 12: Review

- FINAL EXAM: Monday, Dec 12, 2008, 1:00-3:00 PM, in the usual classroom.