History of Mathematics[1]
MA 330, Section 001, Spring 2009

Contents

1 General Information

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Class Time/Location: 8:00-8:50 AM, MWF, Classroom Building 347
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2 Texts

Journey Through Genius: The Great Theorems of Mathematics, by William Dunham. ISBN-10: 014014739X

The Calculus Gallery: Masterpieces from Newton to Lebesgue, by William Dunham. ISBN-10: 0691136262

3 Course Description

We begin with two quotes from John Stillwell.

> One of the disappointments experienced by most mathematics students is that they never get a course on mathematics. They get courses in calculus, algebra, topology, and so on, but the division of labor in teaching seems to prevent these different topics from being combined into a whole. In fact, some of the most important and natural questions are stifled because they fall on the wrong side of topic boundary lines. Algebraists do not discuss the fundamental theorem of algebra because "that's analysis" and analysts do not discuss Riemann surfaces because "that's topology," for example. Thus if students are to feel they really know mathematics by the time they graduate, there is a need to unify the subject.
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> —*Mathematics and its History*
> John Stillwell

> The best way to teach real mathematics, I believe, is to start deeper down, with the elementary ideas of number and space. . . . in fact, arithmetic, algebra, and geometry can never be outgrown. . . by maintaining ties between these disciplines, it is possible to present a more unified view of mathematics, yet at the same time to include more spice and variety.
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> —*Numbers and Geometry*
> John Stillwell

[1] I reserve the right to change or amend this syllabus at any time for any reason.

A course in the history of mathematics provides an opportunity for students of mathematics to remedy the disappointment described by Stillwell. We will think seriously about a variety of the pillars of mathematics, the truly outstanding theorems, while trying to maintain balance and dialogue among the most fundamental branches of mathematics. In this way, we will hopefully create for ourselves a cohesive vision of mathematics and establish connections between mathematics and the non-mathematical world.

Our class sessions will consist of discussions based on daily readings. The readings will be structured around *Journey Through Genius* and *The Calculus Gallery*. There will be a variety of short assignments, from journal responses to traditional problems, and these will often be a starting point for our discussions. There will also be two in-class exams and a cumulative final.

One comment that must be made regarding this course is that there are many possible paths we could take in our investigation of the pillars of mathematics. For example, how does one choose the "best" theorems? What does that even mean? What makes a theorem beautiful? Or useful? Though we will be following the path laid out by the course texts, these questions are important and a large part of our discussions should be dedicated to developing an understanding of our own mathematical aesthetic and how it differs from those of other people. Consider, for example, the following quote.

> *Beauty* and *insight* – these are words that Erdős and his colleagues use freely [in reference to mathematics] but have difficulty explaining. "It's like asking why Beethoven's Ninth Symphony is beautiful," Erdős said. "If you don't see why, someone can't tell you. I know numbers are beautiful. If they aren't beautiful, nothing is."
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> *The Man Who Loved Only Numbers:*
> *The Story of Paul Erdős and the Search for Mathematical Truth*
> Paul Hoffman

While Erdős's thoughts are satisfying in some ways, they fall short in others. There must be reasons why certain theorems are almost universally accepted as profoundly beautiful while others are considered less important. What drives our sense of mathematical value, both individually and collectively? How have those values changed over time? These are perhaps the most fundamental questions to ask in a history of math course because the mathematicians whose work we are studying were inspired by their own values, leading directly to where we are now. So, while we will be reading texts by experts in the history of mathematics, we must look at the topics they have selected with, simultaneously, the utmost respect and a sharply critical eye. The following passage from *Ways of Reading* is very relevant to this point.

> For good reasons and bad, students typically define their skill by reproducing rather than questioning or revising the work of their teachers (or the work of those their teachers ask them to read). It is important to read generously and carefully and to learn to submit to projects that others have begun. But it is also important to know what you are doing – to understand where this work comes from, whose interests it serves, how and where it is kept together by will rather than desire, and what it might have to do with you. To fail to ask fundamental questions – Where am I in this? How can I make my mark? Whose interests are represented? What can I learn by reading with and against the grain? – to fail to ask these questions is to mistake skill for understanding, and it is to misunderstand the goals of a liberal education.
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> *Ways of Reading*
> David Bartholomae and Anthony Petrosky

4 On Reading

> We learned that if our students had reading problems when faced with long and complex texts, the problems lay in the way they imagined a reader – the role a reader plays, what a reader does, why a reader reads (if not simply to satisfy the requirements of a course). When, for example, our students were puzzled by what they read, they took this as a sign of failure. ("It doesn't make any sense," they would say, as though sense were supposed to be waiting on the page, ready for them the first time they read through.) And our students were haunted by the thought that they couldn't remember everything they had read... or if they did remember bits and pieces, they felt that the fragmented text they possessed was evidence that they could not do what they were supposed to do. Our students were confronting the experience of reading, in other words, but they were taking the problems of reading – problems all readers face – and concluding that there was nothing for them to do but give up...
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> Our students need to learn that there is something they can do once they have first read through a complicated text; successful reading is not just a matter of "getting" an essay the first time... You work on what you read, and then what you have at the end is something that is yours, something you made.
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> *Ways of Reading*
> David Bartholomae and Anthony Petrosky

In this course, our central activity will be reading mathematical texts independently. This is not a task that most students are prepared for. Mathematics textbooks are generally viewed by students as databases for homework problems or supplements to lectures rather than as books to be enjoyably read. This is a consequence of the fact that textbooks are often designed and written with conflicting goals in mind: to provide students with homework problems and examples, to provide instructors with flexibility for a variety of courses, to serve students in a diverse range of majors, etc. As a result, while there are some beautiful and fascinating textbooks available, many textbooks are not pleasures to read, which is unfortunate. When a student does stumble upon a textbook that is worth reading, their prior experience often discourages them from investing the time and effort needed to benefit from a serious reading.

The texts we will be reading are not textbooks in the usual sense. They were written to be read, in detail and with joy, by interested readers. They were created for their own purpose, not in an attempt to serve the purposes of others. Because of this, the readings will be challenging; we will need to constantly refine our reading abilities. We need to be careful not to mistake these ordinary challenges for insurmountable obstacles. We also need to recognize, from the beginning, that the act of reading will not get any easier as we go along. Rather, we will become familiar with and accepting of the challenges these texts offer us. The two quotes in this section offer valuable insight regarding reading that we should keep in mind throughout the semester.

> In order to appreciate *[Finnegans] Wake's* reader-friendliness, however, one has to abandon two assumptions about the act of reading which frequently exist side-by-side (though they are, on the surface at least, contradictory). One is that reading is an act of mastery whereby the text is made to yield up all its secrets and allowed to hold nothing back; the other is that reading is a passive experience whereby the reader receives meanings unambiguously communicated by the text. The *Wake* will never be mastered... More than this, however: the *Wake* teaches us, in a most delightful way, that *no* text can be mastered, that meaning is not something solid and unchanging beneath the words, attainable once and for all. All reading, the *Wake* insists, is an endless interchange: the reader is affected by the text at the same time as the text is affected by the reader, and neither retains a secure identity upon which the other can depend.
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> "Reading Joyce," in *The Cambridge Companion to James Joyce*
> Derek Attridge

5 Course Assessment

- *Attendance and Participation*
  - You must be present and engaged in class discussion each day. I will pass around an attendance sheet each day.
  - *WARNING:* Signing another person's name on the attendance sheet will result in an automatic grade of E for the course.
  - Engagement does not mean you have to talk every day, or meet some quota of comments. It means you have to listen to what other people are saying and share your thoughts from time to time. I will try to use the short assignments to facilitate participation, so be prepared for me to ask you to share your responses in class.
  - Your participation grade will be largely subjective. If you have any concerns, please come talk to me.
  - You are allowed 2 unexcused absences. Beyond that, you will lose 2% of your overall course grade for each unexcused absence.
- *Short Assignments*
  - I will assign short assignments on a regular basis. They will take a variety of forms: for example, I might request a written reflection on the reading or I might assign a traditional problem set.
  - I will regularly require typed assignments. Look at the course website for some suggestions on word processing for mathematics.
  - *WARNING:* No late work will be accepted.
- *Exams*
  - There will be two in-class exams and one cumulative final. In cases of excused abscenses as outlined in SR 5.2.4.2 found at http://www.uky.edu/StudentAffairs/Code/, a make-up exam may be arranged. This must be arranged prior to the original exam date and completed within 7 days of the original exam date. Alternate exams may have different problems/questions than those given originally.

6 Course Grades

Your total grade will be determined by your attendance and participation, short assignments, and exams. The grading scale will be no stricter than the usual A>89.9, B>79.9, C>69.9, D>59.9, E otherwise, weighted as follows:

- Attendance and Participation: 10%
- Short Assignments: 30%
- Exams 1 2: 20% each
- Final Exam: 20%

7 Academic Integrity and Classroom Demeanor

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

http://www.uky.edu/USC/New/SenateRulesMain.htm

Turn off all cell phones, pagers, etc. prior to entering the classroom. ***You are not to use your cell phones, pagers, or other electronic devices during class.*** An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

8 Classroom and Learning Accommodations

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.

9 Tentative Schedule

Abbreviations: JTG = Journey Through Genius and TCG = The Calculus Gallery

Jan 14: Syllabus Review

Jan 16: JTG: Preface
HW Due

Jan 19: MLK Holiday

Jan 21: JTG: Ch 1, pgs 1–17
Extending the remarks on page 10 of JTG, familiarize yourself with a proof that $\sqrt{2}$ is irrational that does not use results about decimal expansions. (Wikipedia is one possible source.)
HW Due

Jan 23: JTG: Ch 1, pgs 17–26

Jan 26: JTG: Ch 2, pgs 27–44
HW Due

Jan 28: JTG: Ch 2, pgs 44–53

Jan 30: JTG: Ch 3, pgs 53–68
HW Due

Feb 2: JTG: Ch 3, pgs 68–75
Extending the remarks on page 69 of JTG, familiarize yourself with a proof that the Euclidean algorithm works. (Wikipedia is one possible source.)

Feb 4: Snow Cancellation

Feb 6: JTG: Ch 3, pgs 75–83
HW Due

Feb 9: JTG: Ch 4, pgs 84–99

Feb 11: JTG: Ch 4, pgs 99-112

Feb 13: JTG: Ch 5, pgs 113–132
HW Due

Feb 16: JTG: Ch 6, pgs 133–147

Feb 18: JTG: Ch 6, pgs 147–154
HW Due

Feb 20: JTG: Ch 7, pgs 155–165, TCG: Introduction

Feb 23: JTG: Ch 7, pgs 165–171, TCG: Ch 1, pgs 5–11
HW Due

Feb 25: Review day for Exam 1

Feb 27: Exam 1

Mar 2: JTG: Ch 7, pgs 171–174, TCG: Ch 1, pgs 11-15

Mar 4: JTG: Ch 7, pgs 174–183

Mar 6: TCG: Ch 1, pgs 16–19
HW Due

Mar 9: TCG: Ch 2, pgs 20–29

Mar 11: JTG: Ch 8, pgs 184–190, TCG: Ch 2, pgs 30–34

Mar 13: JTG: Ch 8, pgs 191–199
HW Due

Spring Vacation: March 16–20

Mar 23: JTG: Ch 8, pgs 202–206, TCG: Ch 3, pgs 35–41

Mar 25: TCG: Ch 3, pgs 41–46
HW Due

Mar 27: TCG: Ch 3, pgs 46–51

Mar 30: JTG: Ch 9, pgs 207–212

Apr 1: TCG: Ch 4, pgs 52-60
HW Due

Apr 3: JTG: Ch 9, pgs 212–218

Apr 6: JTG: Ch 9, pgs 218–222, TCG: Ch 4, pgs 60–64

Apr 8: TCG: Ch 4, pgs 65–68
HW Due

Apr 10: JTG: Ch 10, pgs 223–229

Apr 13: JTG: Ch 10, pgs 229–235

Apr 15: Review Day for Exam 2

Apr 17: Exam 2

Apr 20: JTG: Ch 10, pgs 235–244
Following up on JTG page 239, familiarize yourself with a detailed statement of the Fundamental Theorem of Algebra. (Wikipedia is one possible source.)

Apr 22: JTG: 245–258

Apr 24: JTG: 259–266

Apr 27: JTG: 267–273
HW Due VIA EMAIL

Apr 29: JTG: 274–286

May 1: NO CLASS

Final: Final Exam, Tuesday, May 5, 2009; 10:30–12:30