MA 330 ASSIGNMENT
QUADRATURE
DUE WEDNESDAY, SEPT 10

Answers to problems may be handwritten.

Problem 1: In *Journey Through Genius* we are told that $\sqrt{2}$ is irrational. Prove that this is true through the following.

(1) If k is a positive integer, explain why each prime in the factorization of k^2 must occur an even number of times.
(2) Now suppose $\sqrt{2}$ is rational, i.e., $\sqrt{2} = \frac{a}{b}$ for two integers a and b.
(3) Cross-multiply and square to conclude that $a^2 = 2b^2$. Explain why this gives a contradiction, making the expression $\sqrt{2} = \frac{a}{b}$ impossible as stated.
(4) Is there anything special about the number 2 in this argument? For which numbers m does this argument generalize to show that $\sqrt{m}$ is irrational? For which numbers m does this argument fail? Justify your response.

Problem 2: Here is an algebraic version of the quadrature of the lune. Begin with a square of side length $2r$. Upon each side, construct a semicircle. Circumscribe a circle about the original square.

(1) What is the *algebraic* relationship between the area of the original square and the combined areas of the four lunes bounded by the semicircles and the circumscribed circle?
(2) From this, can you conclude that the lune is quadrable? Why or why not?
(3) Is your answer related to Hippocrates' method of quadrature? If so, how? If not, why not?