MA 330 ASSIGNMENT
AN UNSOLVED PROBLEM INVOLVING UNIT FRACTIONS
DUE FRIDAY, SEPTEMBER 26

Here is a problem that heavily involves unit fractions. This is an open, i.e. unsolved, problem given by Jeffrey Lagarias in 2002 in an article in the *American Mathematical Monthly*. Make as much progress as you can on it. Your goal is to do something more than check examples; the examples should lead you to make some interesting observations about the problem, to understand it a bit better. Why do you think it might be true? Why might it be false? Are there any properties of e, H_n, ln, or $\sigma(n)$ that support your comments? Are there special values of n for which this is obviously true? (Seriously, write down everything you're thinking and every idea you try, even if it doesn't go anywhere.)

Open Question: For a positive integer n, let $\sigma(n)$ denote the sum of the positive integers that divide n. For example, $\sigma(4) = 1 + 2 + 4 = 7$, and $\sigma(6) = 1 + 2 + 3 + 6 = 12$. Let H_n denote the nth *harmonic number*, i.e.

$$H_n = \sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}\,.$$

Let ln denote the natural log function. Does the following inequality hold for all $n \geq 1$?

$$\sigma(n) \leq H_n + \ln(H_n)e^{H_n}$$

NOTE: If you successfully answer this question, then the Clay Mathematics Foundation will reward you with $1,000,000. Yes, I'm serious.