MA 330 ASSIGNMENT # 10

Solutions to problems may be handwritten.

Problem 1:

Sylvester's sequence is defined by the recursive formula $s_0 = 2$ and $s_{j+1} = 1 + s_0 s_1 \cdots s_j$. This should remind you of Fermat numbers, which in an earlier homework problem you proved satisfy a similar recursion with a "2" instead of a "1" added on.

Prove that

$$\sum_{j=0}^{\infty} \frac{1}{s_j} = 1\,.$$

NOTE: You may freely use http://planetmath.org/sumofreciprocalsofsylvesterssequence and http://en.wikipedia.org/wiki/Sylvester%27s_sequence to get ideas, but if you use any ideas from these pages you must cite them and explain in your solution how they were used.

Problem 2:

The Fibonacci numbers are the sequence $F_0, F_1, F_2, F_3, \ldots = 0, 1, 1, 2, \ldots$ defined by the recurrence $F_n = F_{n-1} + F_{n-2}$ with the initial values given above.

- Use the defining recurrence for F_n to show that

$$(1 - x - x^2)\left(\sum_{n=0}^{\infty} F_n x^n\right) = x$$

  and hence conclude that

$$\sum_{n=0}^{\infty} F_n x^n = \frac{x}{1 - x - x^2}$$

- Use the equation you just found to prove that

$$\sum_{n=0}^{\infty} \frac{F_n}{2^{n+1}} = 1\,.$$

REMARK: You might find it interesting to compare these two different ways that you can obtain 1 as an infinite series.