

## MA 330 ASSIGNMENT # 12

### Problem 1:

On page 224 in *Journey Through Genius*, it is mentioned that Euler proved Fermat's claim that if a number is of the form  $4k+3$ , then it is not a sum of two squares. Prove Fermat's claim yourself, i.e., prove that a number of the form  $4k+3$  is not expressible as  $a^2 + b^2$  for positive integers  $a$  and  $b$ .

HINT: Consider what happens when you evaluate  $a$  and  $b$  with different combinations of odd and even numbers (i.e., consider this mod 2).

### Problem 2:

Prove that any number of the form  $8k+7$  is not a sum of three squares, i.e.,  $8k+7$  is not equal to  $a^2 + b^2 + c^2$  for any triple of positive integers  $a$ ,  $b$ , and  $c$ .

HINT: If  $x$ ,  $y$ , and  $z$  are integers, what are the possible remainders of  $x^2 + y^2 + z^2$  after division by 8? What happens when you evaluate  $x$ ,  $y$ , and  $z$  using different combinations of numbers of the form  $8n+k$ ? (i.e., consider this mod 8.)

NOTE: In 1770, Lagrange proved that *every number is a sum of four squares*. This is AMAZING, and it is an example of how "high-dimensional" geometry (in other words, having lots of variables when you are doing algebra), can allow more freedom for solving problems than two or three dimensional geometry.