# MA 330 ASSIGNMENT # 2

Answers to problems may be handwritten.

**Problem 1:** In *Journey Through Genius* we are told that $\sqrt{2}$ is irrational. Prove that this is true through the following.

(1) If $k$ is a positive integer, explain why each prime in the factorization of $k^2$ must occur an even number of times.

(2) Now make the assumption that $\sqrt{2}$ is rational, i.e., $\sqrt{2} = \frac{a}{b}$ for two integers $a$ and $b$. Cross-multiply and square to conclude that $a^2 = 2b^2$. Explain why this gives a contradiction to your assumption, implying that $\sqrt{2}$ cannot be written as a fraction $\frac{a}{b}$.

**Problem 2:** Is there anything special about the number 2 in your answer to Problem 1? For which numbers $m$ does this argument extend to show that $\sqrt{m}$ is irrational? For which numbers $m$ does this argument fail? Justify your response.