MA 330 001, History of Mathematics[1]
University of Kentucky, Spring 2015

1. General Information

Instructor: Dr. Benjamin Braun
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Office Phone: 257-6810
Class Time/Location: CB 337, MWF 9:00-9:50AM
Office Location/Hours: POT 831. Meetings in person are by appointment. Online availability via piazza.com, see Online Discussion Board section.

Required Course Texts:

Books:

- *Journey Through Genius: The Great Theorems of Mathematics*, 1991, by William Dunham. ISBN-10: 014014739X
- *The Crest of the Peacock: Non-European Roots of Mathematics*, third edition, 2011, by George Gheverghese Joseph. ISBN-10: 0691135266

Articles:

- "The Secret to Raising Smart Kids," by Carol Dweck, 1 January 2015, Scientific American
  http://www.scientificamerican.com/article/the-secret-to-raising-smart-kids1/
- "Does $1+2+3+4+\cdots$ really equal $-1/12$?," by Evelyn Lamb, 20 January 2014 blog post at Scientific American *Roots of Unity* blog
  http://blogs.scientificamerican.com/roots-of-unity/2014/01/20/is-the-sum-of-positive-integers-negative/
- "Divergent Series," by Ed Sandifer, June 2006 post at Mathematical Association of America *How Euler Did It* column.
  http://eulerarchive.maa.org/hedi/HEDI-2006-06.pdf
- Handout on Euler's proof of infinitude of primes

Videos:

- Khan Academy video on Fermat's Little Theorem
  https://www.youtube.com/watch?v=XPMzosLWGHo
- Numberphile video discussing $1+2+3+4+\cdots=-1/12$
  https://www.youtube.com/watch?v=w-I6XTVZXww

2. Course Description and Student Learning Outcomes

We will begin with three quotes.

> I learned a great deal from taking this course; however, I do not think what I learned was exactly what I was supposed to learn.
>
> Former MA 330 student

> The most important thing I learned in this class is that I have the ability to comprehend things that are very difficult, and have not been taught to me.
>
> Former MA 330 student

[1] I reserve the right to change or amend this syllabus at any time for any reason.

> One of the disappointments experienced by most mathematics students is that they never get a course on mathematics. They get courses in calculus, algebra, topology, and so on, but the division of labor in teaching seems to prevent these different topics from being combined into a whole. In fact, some of the most important and natural questions are stifled because they fall on the wrong side of topic boundary lines... Thus if students are to feel they really know mathematics by the time they graduate, there is a need to unify the subject.
>
> *Mathematics and its History*
> John Stillwell

It isn't at all clear what a course on the History of Mathematics should be. Is it a history course with some math in it? Is it a math course where historical tidbits are tossed in? Should it only cover mathematics that students have already seen, or only mathematics that is new to them, or a blend of both? Should it be an easier course ("the last requirement for my math minor") or a challenging one ("now I really understand Lebesgue integration")?

For what it's worth, I don't have good answers to these questions either. What is more clear is what a History of Mathematics course should do. This course should inform our vision of how mathematics develops, and should allow us to create for ourselves a cohesive picture of mathematics. It should establish connections between mathematics and the non-mathematical world, helping us seek to understand what drives our intellectual values, both individually and collectively. It should cause us to ask deep questions: How have our intellectual values changed over time? What makes a theorem important? What does that even mean? What makes a theorem beautiful? Or useful? What is the role and purpose of proof in mathematics? How do we as mathematical learners and practitioners fit into the contemporary culture of mathematics? These are perhaps the most fundamental questions to ask in a history of math course, because the mathematicians whose work we are studying were inspired by their own answers to these questions.

Motivated by these questions, the student learning goals for MA 330 regard understanding of mathematical content and development of mathematical practices.

Students in MA 330 will deepen their understanding of

(1) Greek, Egyptian, and Mesopotamian mathematics, with a focus on arithmetic, number theory, Euclidean geometry, and rhetorical algebra;
(2) how the Islamic mathematicians blended these mathematical traditions with traditions from the Indian subcontinent;
(3) how algebra and calculus were initially regarded and practiced, and how this differs from contemporary use;
(4) the development of mathematical approaches to infinite processes; and
(5) the role, purpose, and development of mathematical proof.

Further, students in MA 330 will

(6) enhance their reading, writing, and oral communication skills in mathematical contexts;
(7) increase their persistence and use of self-monitoring when working on mathematics;
(8) develop an understanding of the global nature of mathematical culture, and the importance of cultural interactions in mathematical history; and
(9) reflect on contemporary mathematical culture, their place in it, and their mathematical values.

It is important to remember that we cannot reach these learning outcomes completely on our own, as shown by the following quote.

> Within these [mathematical] communities of practices, there are well-established patterns of behaviour to which I have drawn attention [in this study]. The most important is that mathematics is no longer seen, by the majority of mathematicians, as an individual activity and all of the reasons which [the mathematicians in this study] gave for collaborating can be found in the education literature with respect to utilizing group work in classrooms. The experiences of these mathematicians help, I think, to emphasize the flow and inter-dependence in meaning making between the socio-cultural formulations and individual acquisition. Learning is *neither* wholly individual *nor wholly* social.
>
> *The Practices of Mathematicians*
> Leone Burton

3. **Class Structure**

Our activities in this course will directly serve the student learning outcomes listed above. Class time will usually not be spent in a formal lecture style. Instead, we will spend time in class:

- discussing the readings with the entire class,
- working through math in the readings in groups,
- working on problem sets related to the readings in groups,
- presenting problem progress and solutions at the board, and
- offering and receiving constructive criticism regarding our ideas and understanding.

You should expect to spend *at least* six hours per week outside of class for MA 330.

4. **Online Discussion Board**

MA 330 Online Discussion Board:

https://piazza.com/uky/spring2015/ma330/home

Piazza is an online discussion board where we can collectively ask and answer questions. While you are expected to participate on the discussion board, *the mathematical content on the discussion board is not part of your course grade.* The purpose of the discussion board is to allow us a place where we can make mistakes, introduce errors, head down blind alleys, etc, as a community of learners in a common search for understanding. When you are asking questions, answering questions, giving partial answers to questions, suggesting ideas, etc, please keep at the front of your mind the following quote.

> Learning is least useful when it is private and hidden; it is most powerful when it becomes public and communal. Learning flourishes when we take what we think we know and offer it as community property among fellow learners so that it can be tested, examined, challenged, and improved before we internalize it.
>
> *Taking Learning Seriously*
> Lee Shulman

5. **Tentative Schedule**

Next to each date is listed the material that should be read prior to class. We use the abbreviations JTG = Journey Through Genius and Crest = The Crest of the Peacock.

Jan 14: First Day of Class
Jan 16: JTG: Preface
Crest: Preface to the First Edition
Dweck article "Secret to Raising Smart Kids": read all of it
Course Syllabus
ASSIGNMENT #1 DUE

Greek Mathematics

Jan 19: No Class: MLK Holiday
Jan 21: JTG: Ch 1, pgs 1–17, start chapter, end at "Great Theorem"
Jan 23: JTG: Ch 1, pgs 17–26, start "Great Theorem," complete chapter.
ASSIGNMENT #2 DUE
Jan 26: JTG: Ch 2, pgs 27–44, start chapter, end at "Book I: Parallelism..."
EXAM
Jan 28: JTG: Ch 2, pgs 44–53, start at "Book I: Parallelism," end at "Epilogue"
Jan 30: JTG: Ch 3, 61–75, start chapter, end at "The Final Books..."
ASSIGNMENT #3 DUE
Feb 2: JTG: Ch 3, 75–83, start at "The Final Books...," complete chapter
EXAM
Feb 4: JTG: Ch 4, pgs 84–99, start chapter, end at "Archimedes' Masterpiece"
*** Course Project Proposal Due ***
Feb 6: JTG: Ch 4, pgs 99–112, start at "Archimedes' Masterpiece," complete chapter
ASSIGNMENT #4 DUE

Egyptian and Mesopotamian Mathematics

Feb 9: Crest: Ch 1
EXAM
Feb 11: Crest: Ch 3, pgs 79–100, start chapter, end at "Applications of Unit Fractions"
Feb 13: Crest: Ch 3, pgs 100–109, start at "Applications of Unit Fractions, end at "Egyptian Geometry"
Crest: Ch 3, pgs 119–122, subsection on "Egyptian Mathematics: A General Assessment"
ASSIGNMENT #5 DUE
Feb 16: Crest: Ch 4, pgs 125–144, start chapter, end at "A Babylonian Masterpiece"
EXAM
Feb 18: Crest: Ch 4, pgs 150–159, start at "Babylonian Algebra," end at "Babylonian Geometry"
Feb 20: Crest: Ch 5

Focus on Course Projects

Feb 23: No reading
Feb 25: PEER EDIT DAY: First Version of Course Project Due

Islamic Mathematics

Feb 27: JTG: Ch 5, pgs 129–132, start in Epilogue after proof of Pythagorean Theorem, complete chapter
Crest: Ch 11, pgs 450–469, start chapter, end at "Mathematics in the Service of Islamic Law"
ASSIGNMENT #6 DUE

Mar 2: Crest: Ch 11, pgs 471-474, start at "The Theory of Numbers," end at "Extraction of Roots"
EXAM

Mar 4: Crest: Ch 11, pgs 475–487, start at "Extraction of Roots," end at "Geometry in the Islamic World"

Mar 6: Crest: Ch 11, pgs 487–492, start at "Geometry in the Islamic World," end at "Thabit ibn Qurra's Generalization"
Crest: Ch 11, pgs 508–512, start at "The Islamic Contribution: Final Assessment," complete chapter
ASSIGNMENT #7 DUE

The Beginning of symbolic algebra

Mar 9: JTG: Ch 6, pgs 133–147, start chapter, end at "Further Topics on Solving Equations"
EXAM

Mar 11: JTG: Ch 6, pgs 147–154, start at "Further Topics on Solving Equations," complete chapter

Mar 13: No Reading
PEER EDIT DAY: First Version of Essay from Assignment #8
ASSIGNMENT #8 DUE

Spring Break: March 16–20 – no class

Infinite Series and the Origins of Calculus

Mar 23: JTG: Ch 7, pgs 155–165, start chapter, end at "Newton's Binomial Theorem"
EXAM

Mar 25: JTG: Ch 7, pgs 165–174, start at "Newton's Binomial Theorem," end at "Great Theorem"

Mar 27: JTG: Ch 7, pgs 174–183, start at "Great Theorem," complete chapter
ASSIGNMENT #9 DUE

Mar 30: JTG: Ch 8, pgs 184–196, start chapter, end at "Great Theorem"
EXAM

Apr 1: JTG: Ch 8, pgs 196–206, start at "Great Theorem," complete chapter

Leonard Euler, "Master of Us All"

Apr 3: JTG: Ch 9, pgs 207–212, start chapter, end at "Great Theorem"
ASSIGNMENT #10 due

Apr 6: JTG: Ch 9, pgs 212–222, start at "Great Theorem," complete chapter
EXAM

Apr 8: Handout regarding Euler's proof of infinitude of primes

Apr 10: No Reading
ASSIGNMENT #11 DUE

Apr 13: Read blog posts by Lamb and Sandifer – *watch the Numberphile video that is embedded in the post by Lamb*
EXAM

Apr 15: JTG: Ch 10, pgs 223–229, start chapter, end at "Great Theorem"

Apr 17: JTG: Ch 10, pgs 229–244, start at "Great Theorem," complete chapter

Apr 20: Watch Khan Academy video on Fermat's Little Theorem
EXAM
ASSIGNMENT # 12 due

Completion of Course Projects

Apr 22: No Reading
Apr 24: Final Version of Course Project Due

The Nature of Infinity

Apr 27: JTG: Ch 11, pgs 245-258, start chapter, end at "Great Theorem"
EXAM
Apr 29: JTG: Ch 11, pgs 259–266, start at "Great Theorem," complete chapter
May 1: No Reading
ASSIGNMENT # 12 due

Final Exam and Discussion Time

Final: 8:30AM–10AM, Monday, May 4, 2015
NOTE: Our final session will last 90 minutes, and will take place from 8:30AM–10AM.
There will be a final exam lasting approximately 30 minutes, followed by a one-hour final class discussion.

6. Assessment and Grading

There will be four elements to assessment and grading in this course: Participation, Homework Assignments, Exams, and a Course Project.

6.1. Participation and Attendance.

- You must be present and engaged in class each day.
- You are expected to participate on the course discussion board.
- Your participation grade will be largely subjective. If you have any concerns, come talk with me about them.
- *Absence Policy:*
  - You are allowed 3 unexcused absences. Beyond that, *you will lose 2% of your overall course grade for each unexcused absence.*
  - Students must notify the instructor of their absence prior to the absence or within one week after the absence.
  - Students must submit any written documentation supporting their excused absence within one week after the absence.
  - Absences for major religious holidays require one week advance written notification.

6.2. Homework Assignments.

- Assignments will be given regularly. Some portions of the assignments must be typed.
- *WARNING:* No late work will be accepted.
- You should work with other students and share your ideas as part of our course community. However, you should not let your collaboration devolve into letting someone else do all the "hard parts" and then copying their answers.
- *Four Rules for Assignments:*
  - Don't talk to anyone about the problems until you have made a genuine effort to solve them yourself.
  - You must write up the solutions on your own.
  - For each problem, write the names of any other people (students, tutors, etc) with whom you shared ideas.

  - You may *not* search the internet for solutions to problems. We will use our creativity, course texts, and peer collaboration as our tools for investigating the history of mathematics.

6.3. **Exams.**

- Exams will take place roughly once per week, at the start of class.
- Exam dates are listed on the course schedule.
- Most exams will be approximately 10-15 minutes in length.

6.4. **Course Project.**

- You will choose a topic for and complete a major project related to the history of mathematics during the course of the semester. This will be a written project of length 10 pages (without cover sheet or references) with 1 inch margins, 12 point Times New Roman font, double spaced. The main requirement is that your project must involve a "great idea" of mathematics and provide a well-supported argument justifying this choice of topic. All projects are expected to be well-written, free from grammatical errors, and have excellent mathematical depth and style. A grading rubric will be provided early in the semester.
- You should direct a significant portion of your project toward a general university audience and articulate clearly which sections are aimed toward experts. *Journey Through Genius* is a good model for this type of exposition.
- You will turn in a first version of your project for peer review; the first version must be a complete project that you will revise substantially to create your final version.

6.5. **Course Grades.** Your course grade will be determined by your attendance, participation, assignments, exams, and project. The grading scale will be no stricter than the usual A>89.9, B>79.9, C>69.9, D>59.9, E otherwise, weighted as follows:

- Participation: 5%
- Assignments: 30%
- Exams: 30%
- Project:
  - First Version: 10%
  - Final Version: 25%

7. Course Expectations and Classroom/Learning Accomodations

All students are expected to follow the academic integrity standards as explained in the University Senate Rules, particularly Chapter 6, found at:

http://www.uky.edu/Faculty/Senate/rules_regulations/index.htm

Turn off all cell phones, pagers, etc. prior to entering the classroom. ***You are not to use your cell phones, pagers, or other electronic communication devices during class.*** An attitude of respect for and civility towards other students in the class and the instructor is expected at all times.

Any student with a disability who is taking this course and needs classroom or exam accommodations should contact the Disability Resource Center, 257-2754, room 2 Alumni Gym, jkarnes@uky.edu.

8. Graduation Writing Requirement Information

Learning Outcomes. This is a writing-intensive (W) course approved to fulfill the upper tier of the graduation writing requirement (GWR).

- Write a paper that is essentially free of mechanical errors (grammar, punctuation, spelling, and syntax) and awkwardness, using a style that is appropriate to the purpose and audience.
- Demonstrate an ability to discover, evaluate, and clearly present evidence in support of an argument in the subject area and utilize documentation that conforms to the formats and the citation conventions of the subject area.
- Be aware that composing a successful text frequently takes multiple drafts, with varying degrees of focus on generating, revising, editing, and proofreading.
- Write a capable, interesting essay about a complex issue (discipline-specific) for a general university audience.

Minimum Writing Requirements.

- Students will be required to write a minimum of 15 pages of formal writing.
- At least 10 of these pages must be single-authored assignments.
- No assignments requiring fewer than 4 pages may be included in the 15-page minimum.
- These 15 pages must go through a draft, review, and revision process. Peer review is sufficient to meet the review requirement.

Grading Policies. To pass the course, students must earn a grade of C or higher on ALL FORMAL assignments. Instructors can consider additional formal writing, writing other than the formal writing, or additional projects and assignments in the final grade computation. Thus, students can receive lower than a C as a final grade and still receive GWR credit. Any major assignment that receives a D or below must be revised to reflect competency and resubmitted. Instructors may limit the number of revision attempts and set time restrictions on revisions. At the discretion of the instructor, students who fail to achieve competency may receive an I (incomplete) grade, but in no case may a student whose writing fails to reach the level of C (competent) receive a passing grade in a course that satisfies the University Writing Requirement.

Plagiarism. Part II of Student Rights and Responsibilities (6.3.1; online at http://www.uky.edu/StudentAffairs/Code/part2.html) states that all academic work written or otherwise submitted by students to their instructors or other academic supervisors is expected to be the result of their own thought research or selfexpression. In cases where students feel unsure about a question of plagiarism involving their work they are obliged to consult their instructors on the matter before submission.

When students submit work purporting to be their own but which in any way borrows ideas organization wording or anything else from another source without appropriate acknowledgment of the fact the students are guilty of plagiarism.

Plagiarism includes reproducing someone else's work whether it be published article chapter of a book a paper from a friend or some file or whatever. Plagiarism also includes the practice of employing or allowing another person to alter or revise the work which a student submits as his/her own whoever that other person may be. Students may discuss assignments among themselves or with an instructor or tutor but when the actual work is done it must be done by the student and the student alone.

When a student's assignment involves research in outside sources or information the student must carefully acknowledge exactly what where and how he/she has employed them. If the words of someone else are used the student must put quotation marks around the passage in question and add an appropriate indication of its origin. Plagiarism also includes making simple changes while leaving the organization content and phraseology intact. However nothing in these Rules shall apply to those ideas which are so generally and freely circulated as to be a part of the public domain.