8/29/2016

1. Mathskeller + Study now open for tutoring – See Canvas announcement.

2. Webwork A1 and A2 due this week. WA #1 due Friday. See Canvas.

3. Quiz #1 Thursday. See Canvas.

Q: With neighbors: Why do we define $2^0 = 1$? Does this make sense? Why or why not?

Q: (a) If a population of bacteria doubles every hour and has a starting population of 162 bact., how many bact. are there after 5 hours? after 5.25 hours?

A: (a) after t hours, Let population be denoted $P(t)$. Then

$$P(t) = 162 \cdot 2^t$$

$$\boxed{2^{5.25} = 2^{5+1/4} = 2^5 \cdot 2^{1/4}}$$

So, $P(5) = 162 \cdot 2^5 = 5{,}184$ bact.

$$P(5.25) = 162 \cdot 2^{5.25} = 162 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2^{1/4}$$

$$(5.25 = 5 + 1/4) \quad = 5{,}184 \cdot \sqrt[4]{2}.$$

Aside

Why is $2^{1/4} = \sqrt[4]{2}$?

$$2 = 2^1 = 2^{1/4+1/4+1/4+1/4} = 2^{1/4} \cdot 2^{1/4} \cdot 2^{1/4} \cdot 2^{1/4}$$
$$= \left(2^{1/4}\right)^4$$

When you raise $2^{1/4}$ to the 4^{th} power, you get 2.

So, $2^{1/4} = \sqrt[4]{2}$. (warning! there are 4 4^{th} roots of 2.)

Q: (b) With the same setup, $P(t) = 162 \cdot 2^t$, at what time do we have 10,000 bacteria?

Need to solve: $10{,}000 = 162 \cdot 2^t$

$$\Rightarrow \frac{10{,}000}{162} = 2^t$$

$$\Rightarrow \log_2\left(\frac{10000}{162}\right) = t \longrightarrow \approx 5.94\ldots \text{ hours}$$

Let's remind ourselves of general setup:

Fix $b>0$. Set $f(x)=b^x$.

- If x is a positive integer, repeatedly multiply. eg. $b^6 = b \cdot b \cdot b \cdot b \cdot b \cdot b$
- If x is a fraction, you multiply and take roots.

  e.g. $b^{6.7} = b^{67/10} = \sqrt[10]{b^{67}}$
- If x is irrational, you approximate by a rational #. eg. $\pi \approx 3.14$

  $b^{\pi} \approx b^{3.14} = b^{314/100} = \sqrt[100]{b^{314}}$

__Note__: For $b>0$, b^x is 1-1, so it has an inverse called $\log_b(x)$.

__Rule__: $y = b^x$ when $\underbrace{x = \log_b(y)}_{\text{(very subtle)}}$.

__Read in book__:
- Domain, range of b^x + $\log_b(x)$
- Laws of Exponents
- Laws of Logs.

__"Natural" logs and e.__

$\sqrt{2}$, 

$e \approx 2.71828\ldots$. And it has excellent properties.

e^x is the base with the property that

$$e^x \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{3 \cdot 2} + \frac{x^4}{4 \cdot 3 \cdot 2} + \frac{x^5}{5 \cdot 4 \cdot 3 \cdot 2} + \ldots \text{ etc.}$$

Follow this pattern as far as you want, to get more and more accurate approximations.

<u>NOTE</u>: We write $\log_e(x) = \ln(x)$.

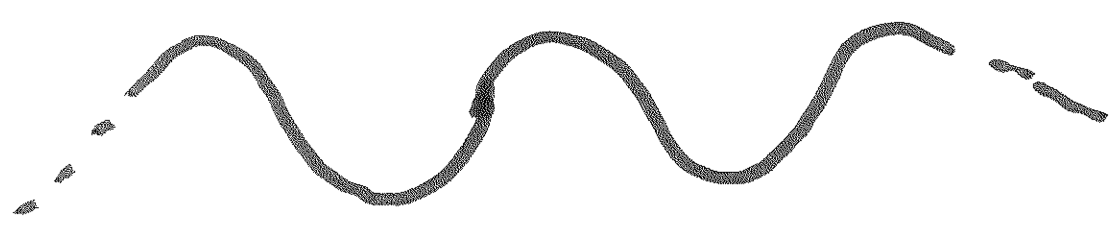
8/31/2016

$\boxed{1}$ Please review today's Canvas announcement regarding Exams.

$\boxed{2}$ Discuss with your neighbors:

(a) What is the definition of $\sin\theta$?

(b) Why does the graph of $\sin\theta$ look like



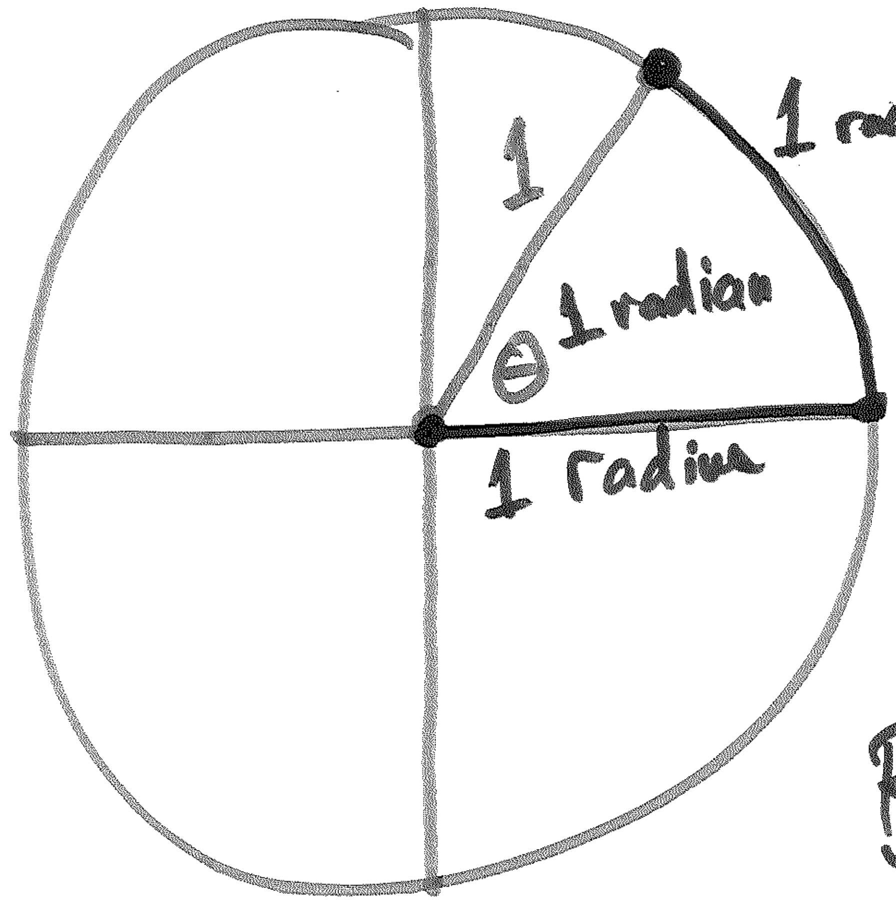
What is def$^{\underline{n}}$ of $\sin\theta$? Student responses

- opp/hyp
- y-coord on unit circle
- pos in quad 1 + 2.

What images come to mind when you think of $\sin\theta$?

- right triangle
- wave, i.e. graph
- unit circle
- sun rising

Start with unit circle:



1st consider angle θ.

How do we measure θ <u>Mathematically?</u>

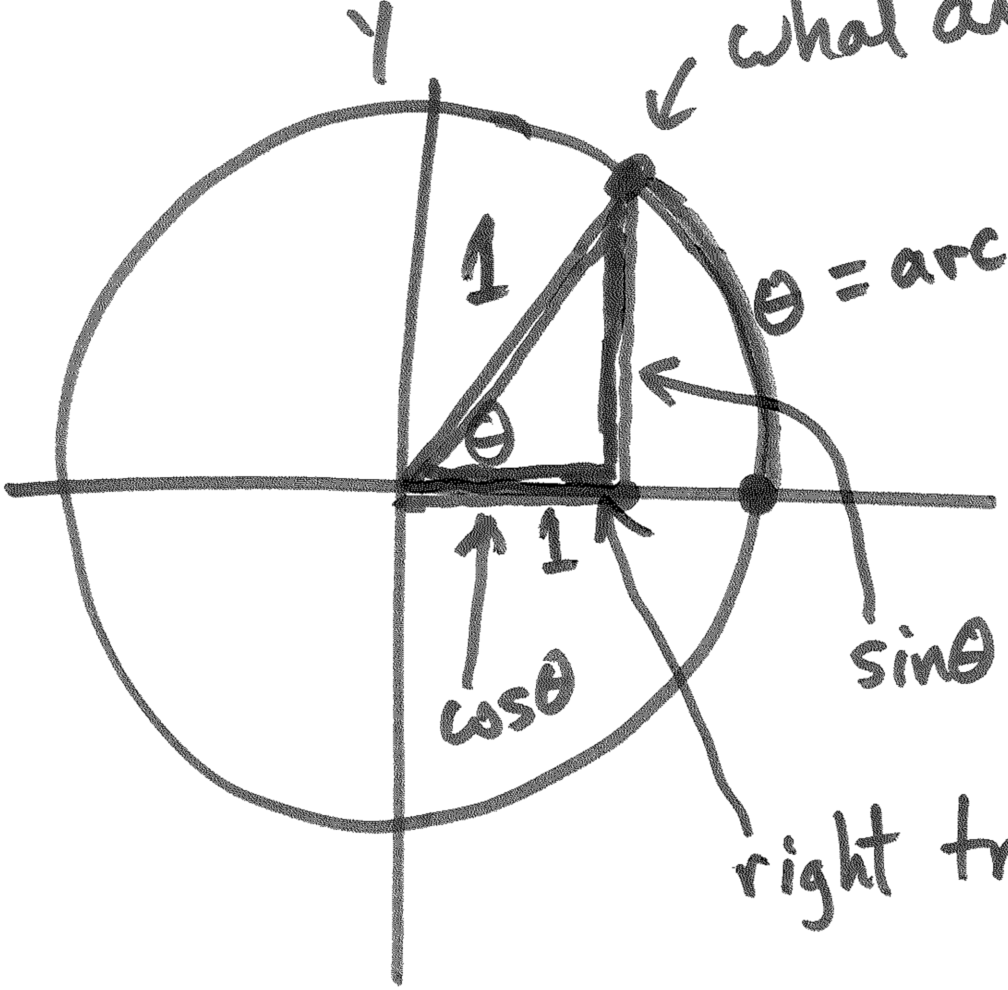
<u>Radial</u> <u>Angles</u>

Radian

See Wikipedia for picture.

What about sin + cos? What are they?

what are x- and y-coords here?



θ = arc length in angular region.

We define $\sin\theta$ and $\cos\theta$ to be these x- and y-coords.

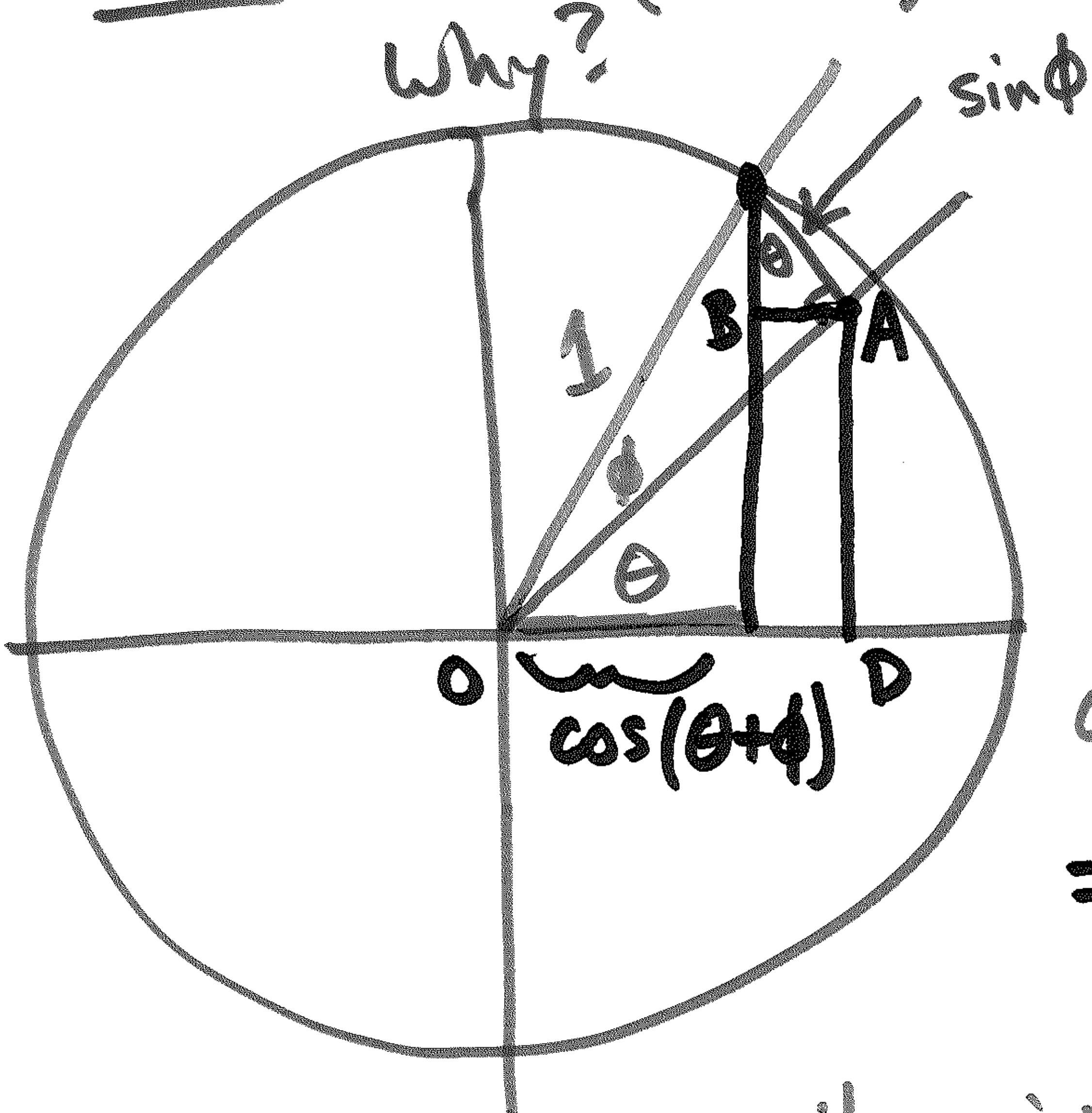
right triangle: $\dfrac{opp}{hyp} = \dfrac{\sin\theta}{1} = \sin\theta$

$\dfrac{adj}{hyp} = \dfrac{\cos\theta}{1} = \cos\theta.$

Why do $\sin\theta$ + $\cos\theta$ have their graphs? See Wikipedia for trig functions.

Q: How else can this help us understand?

Ex: $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$



Show using similar $\triangle$'s that:

$OD = \cos\theta\cos\phi$

$AB = \sin\theta\sin\phi$

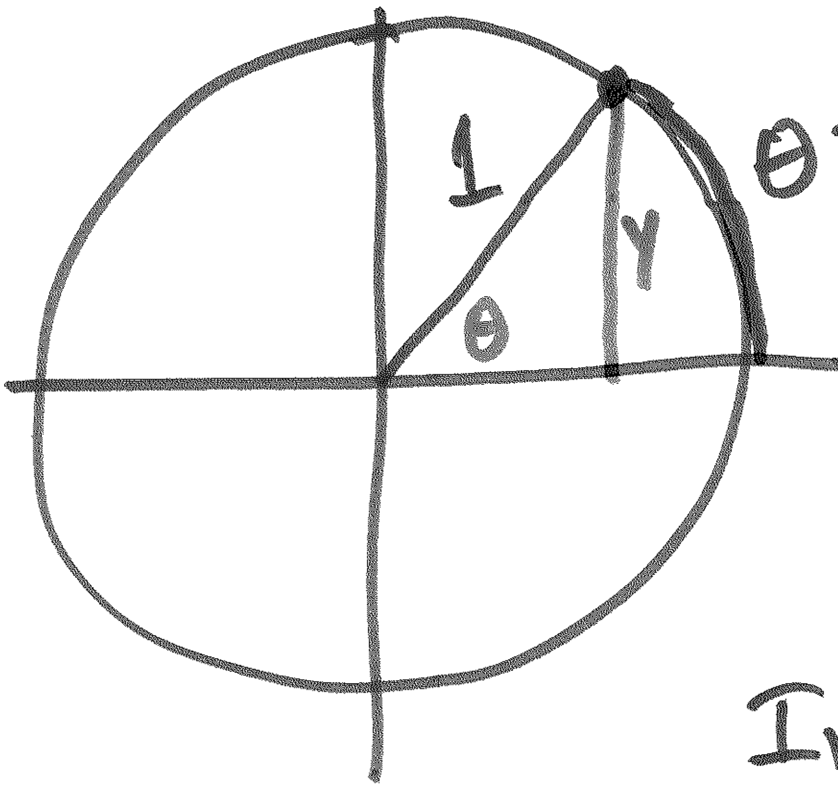
$\cos(\theta+\phi) = OD - AB$

$= \cos\theta\cos\phi$

$\quad - \sin\theta\sin\phi.$

Similar picture for $\sin(\theta+\phi)$.

Inverse trig:



$\theta =$ arc length

By definition,

$y = \sin\theta$.

$\theta =$ arc length $\longmapsto y = \sin\theta$ line length.

Inverting this gives us

$y =$ line length $\longmapsto \theta = \sin^{-1} y$

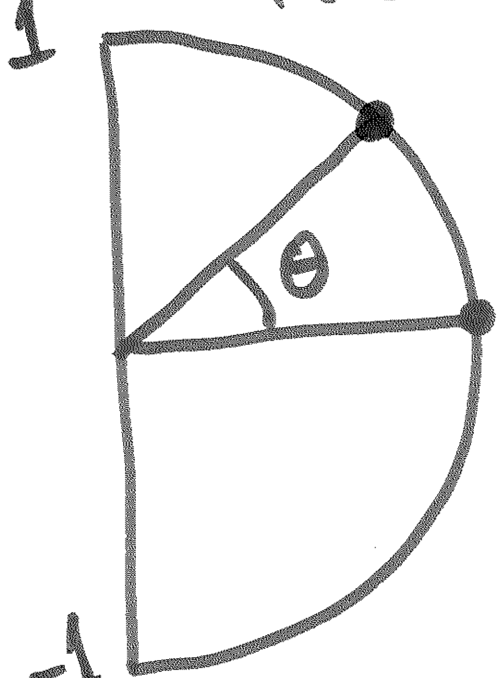
$= \arcsin y$

$=$ length of arc.

$\sin\theta$ is not 1-1 on $(-\infty,\infty)$.
So, we restrict the domain to make $\sin\theta$ invertible.

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $-1 \le y \le 1$.

for $\arcsin y$ function.



Ex: $\arcsin(\sin(\pi))$
$= \arcsin(0) = 0$.

See §1.5 for cos, tan, etc.